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ANALYTIC SIMULATION OF THE PERFORMANCE OF MOBILE MAINTENANCE CONTACT TEAMS

GEORGE J. SCHLENKER

MAY 1985

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REPORT DOCUMENTATION PAGE

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1. REPORT NUMBER AMSMC/RD/MR-6	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) Analytic Simulation of the Performance of Mobile, Maintenance Contact Teams		5. TYPE OF REPORT & PERIOD COVERED Report - Final 6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) George J. Schlenker		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS US Army Armament, Munitions & Chemical Command Readiness Directorate Rock Island, IL 61299-6000		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS		12. REPORT DATE May 1985 13. NUMBER OF PAGES 53
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)		15. SECURITY CLASS. (of this report) UNCLASSIFIED 15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

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18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Operations Research
Queueing Theory
Markov Processes
Reliability & Maintainability
Maintenance Systems
Contact Teams
Numerical Methods
Distribution Theory

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This report presents an approach to describing the performance of mobile, maintenance contact teams (C-teams). These teams support a tactical military system within a specific service area, such as along a Division Front. The teams are dispatched to a failed unit, diagnose and repair the fault, and move to the next customer without leaving the service area. The mathematical model of this system describes the stochastic steady-state using Markov process theory. Solutions are obtained by solving the linear, steady-state equations (continued)

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20. ABSTRACT (continued):

using an efficient numerical procedure. The implementing computer program source code is included in the report.

A parametric analysis was performed using values characteristic of an air defense system. Parameters examined include: (a) intercustomer speed and variation in speed, (b) density of the C-teams, (c) dedicated versus nondedicated teams, (d) mean time between service requests, (e) mean diagnostic and repair time, and (f) form of the probability distribution for total service time.

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EXECUTIVE SUMMARY

This report presents an approach to describing the performance of mobile, maintenance contact teams (C-teams). These teams support an operational system within a specific service area. In the interest of generality, the supported system is left undefined. However, values selected for a parametric analysis were based on a tactical air defense system. The ultimate objective of C-team service is to keep the customer system in a high state of operational readiness. Thus, C-team performance is measured by variables such as average number of nonoperational customer units and average time a failed unit stays nonoperational. Although the mathematical model of the system in steady-state is fully analytic, it is not an expected-value model. The probability distribution for number of nonoperational customer units is an output of the implementing computer program. Statistics summarizing the performance of- and utilization of C-teams are also outputs.

A parametric analysis was performed to examine the sensitivity of C-team performance to parameters such as: (a) intercustomer speed and variation in speed, (b) density of C-teams, (c) dedicated versus nondedicated C-teams, (d) mean time per unit between service requests, (e) mean diagnostic and repair time, and (f) form of the probability distribution for total service time. Some conclusions are drawn from these results, which may be applicable to an operational system of interest. For example, when each of 3 C-teams is dedicated to 1/3 of the customer population, a substantial performance penalty is paid relative to an arrangement of nondedicated C-teams, which serve the entire population.

For the interested analyst, the report sketches the method of derivation of formulas. Computer source code for the implementing programs is included with extensive comments.

CONTENTS

AMSMC-RDA-S May 1985

MEMORANDUM REPORT

SUBJECT: Analytic Simulation of the Performance of Mobile Maintenance Contact Teams

1. References

a. Memorandum Report No DRSMC/SA/MR-4, (AD A136580), HQ, AMCCOM, Nov 83, title: Models of a Service System for Production Machine Maintenance.

b. Technical Report, Society for Industrial and Applied Math, 1979, title: LINPAC Users' Guide.

c. Textbook by Donald Gross and Carl Harris, J. Wiley, c. 1974, title: Fundamentals of Queueing Theory.

2. Background

A maintenance contact team (C-team) is an adjunct to Organizational- and/or DS- maintenance. Each C-team is considered a mobile service system which operates within a prescribed service area. The C-team concept treated here is a derivative of the radar/electronics van supporting the Vulcan Air Defense System, which was developed in the 1960's. A C-team is dispatched so that it can move to the next customer immediately upon completing service on the current customer. In addition to the time to diagnose and repair a fault, the total time a C-team "serves" a customer includes the time to travel to the customer, following a commitment to that customer. Intercustomer distance and speed are variables which affect the service performance. Variations in these random variables as well as the conventional variation in maintenance service time are factors considered in modeling this service system. Performance of this service system is measured by the operational availability of the system being supported. To be general, the latter system is not specified here.

3. Objectives

An objective of this MFR is to describe an analytic -- as opposed to Monte-Carlo -- model which simulates the stochastic steady state of a service system consisting of (possibly) multiple C-teams serving a finite population of customers within a service area. A secondary objective is to present some parametric results which may be helpful in describing the adequacy of C-team performance. Parameter values used in this analysis are based on a range of values considered applicable to the SGT York air defense system. The relative importance of certain parameters and insensitivity of others are displayed.

4. Model Assumptions

The customers are located with uniform probability density in a rectangular service area. Contact teams are dispatched from one customer to the next without leaving the service area. When a C-team completes service on one customer, it either (a) immediately travels to the next customer, if a customer is waiting, or (b) waits at current position until dispatched, if the queue of customers is empty. The path traveled by C-teams between customers follows a series of segments, each of which is parallel to one of the sides of the service area. (The last assumption represents use of road networks and the need to avoid obstructions.) The intercustomer speed varies randomly over occasions with a uniform probability density between prescribed limits. The next customer to be served is selected by means of one of two service disciplines -- FIFO or SDST. Using FIFO, the next customer is the one whose request came first. Using SDST, the next customer is the one with the shortest distance from the team's current position. All prametric results shown here use FIFO service discipline. (When the average queue length is small, there is no practical difference in results from the two disciplines.) The requests for C-team service are assumed to occur at random in a conditionally Poisson manner. The rate parameter per operational unit is a constant. Thus, arrival rate of requests is proportional to the number of operational units, i.e., units not down for C-team maintenance. Disabling faults other than those repairable by C-teams are not treated explicitly in this model. The diagnostic and repair function is always assumed to restore a failed unit to an operational status. The average time to do this is the MTTR. The nominal values of the variables used in this study are given in Table 1. Parametric excursions were made for selected variables.

TABLE 1

VARIABLES USED IN THE CONTACT-TEAM PARAMETRIC ANALYSIS

Standard Parameter Values	
Frontal Width of Service Area	20 km *
Cross-front Dimension of Area	4 km
Intercustomer Average Speed	20 km/hr
Range of Uniformly Dist'd Speed	+- 5 km/hr
Avg Time Between Service Requests	100 hr/unit
Avg Diagnotic and Repair Time	3 hr
Population Dens of Fire Units	12 to 36
Number of Supporting C-Teams	1 to 3

* Front of service area represents a Division Front.

5. Methodology

Every aspect of this problem has been approached analytically, and has been verified by Monte-Carlo methods. This includes the model of C-team movement between customers as well as the stochastic service or queueing model. One advantage of a purely analytic approach is that parametric analyses can be performed with assurance that the observed differences in output statistics are in no way affected by Monte-Carlo sampling variation. This aspect is particularly important when the natural variation in subject variables is relatively large or when differential parametric effects are small. Altho some of the model assumptions may seem somewhat restrictive -- such as a uniform distribution of customers within the service area, these have, in fact, been found not to be so. Some alternatives, which are not analytically tractable, have been examined via Monte Carlo. These auxiliary studies are not reported here.

6. The service performed by a C-team has two stages -- travel to a customer and repair of a fault. Therefore, it is appropriate to consider as a model of the service system a Markov model having two stages of service and serving a finite customer population. The total time to "serve" a customer is considered a gamma(2) random variable. This model is equivalent to two identical stages of exponential service in tandem. Alternative models of service were also used. These are discussed under "Results". A multi-server analytic model of this sort was used earlier (Ref a.) to represent the service system for production machine maintenance. Numerical methods are used to solve the steady-state equations, which derive from the Kolmorogov differential-difference equations for the state probability vector. In the present application, it was considered necessary to change the implementing computer program in order to reduce execution time for cases having a large customer population. Program changes responsible for a great reduction in CPU time are: (a) use of a better method for solving the state equations and (b) reduction in the number of Markov states. In the computer program, found in Annex B, the reduced state-transition matrix, derived from the Kolmogorov equations, is, first, factored (into upper and lower triangular matrices), and then inverted in place. Two LINPAC routines (Ref b.) are used for this purpose. This approach is about 2.5 times faster than the gaussian-reduction method of Ref a. for large (g.t. 100 X 100) matrices. The dimension of the state probability vector is reduced by eliminating states with negligible probability of being occupied. The choice of states to eliminate is based on the closed-form solution to a similar queueing problem. The similar problem involves a finite customer population with conditionally Poisson arrivals and exponential services, whose steady-state solution is found in Reference c. The deleted states are those associated with a number in the service system greater than N, where the probability that the system number exceeds N is smaller than some small value (acceptable error). An error probability of 1/10,000 generally reduces the number of states by more than a factor of 1/2 for the cases treated here. This reduction is quite significant, since CPU time is nearly proportional to the number of states squared.

7. Survey of Results

Several formulas of interest were derived for this model. The system performance requires inputs to the queueing model such as: (a) average time a C-team spends in travel between customers and (b) average maintenance service time per customer. The first variable depends upon the mean and variance of the intercustomer distance and the mean and variance of the intercustomer speed. Formulas were derived for the probability distribution of intercustomer distance and for the mean and variance of this distribution. Since the intercustomer speed is considered a uniform random variable over the range (s1, s2), the mean and variance of the speed are the standard results: (s1 + s2)/2 and (s2 - s1)**2/12, respectively. An approximation for the mean and variance of the ratio of two random variables can be used to obtain the mean and variance of the intercustomer travel time -- = distance/speed -- in terms of the means and variances of distance and speed. The accuracy of the approximation depends somewhat upon the skewness of the distributions of distance and speed. The approximation was checked against an exact method for a practical range of values of the distance and speed parameters and found to be quite adequate. The approximation can be given, simply, as follows. Let E(d) and V(d) be the mean and variance of the intercustomer distance. Similarly, let E(s) and V(s) be the mean and variance of the speed. Then, the mean and variance of the travel time are given, respectively, by

$$E(t) = E(d)/E(s) + E(d)V(s)/(E(s))^3$$

and

$$V(t) = V(d)/(E(s))^2 + V(s)(E(d))^2/(E(s))^4 + (E(d)V(s))^2/(E(s))^6 + V(d)V(s)/(E(s))^4 \, .$$

Equations for the mean and variance of the intercustomer distance for a FIFO service discipline, are given in terms of the dimensions of the rectangular service area, say, a by b:

$$E(d) = (a + b)/3$$

and

$$V(d) = (a^2 + b^2)/18 \, .$$

Equations for the cumulative distribution function (C.D.F.) of intercustomer distance are presented in Annex A. The method of derivation is outlined in this annex. Annex B contains a computer source listing of the program which implements all the auxiliary equations, such as those above, as well as the equations of the Markov queueing model. An example of the C.D.F. of intercustomer distance is given in Figure 1 using the nominal dimensions of the service area. The C.D.F. of intercustomer distance using a SDST service discipline, given n customers are queued (n g.t. 0), is also shown in Figure 1. Notationally, let the latter C.D.F.

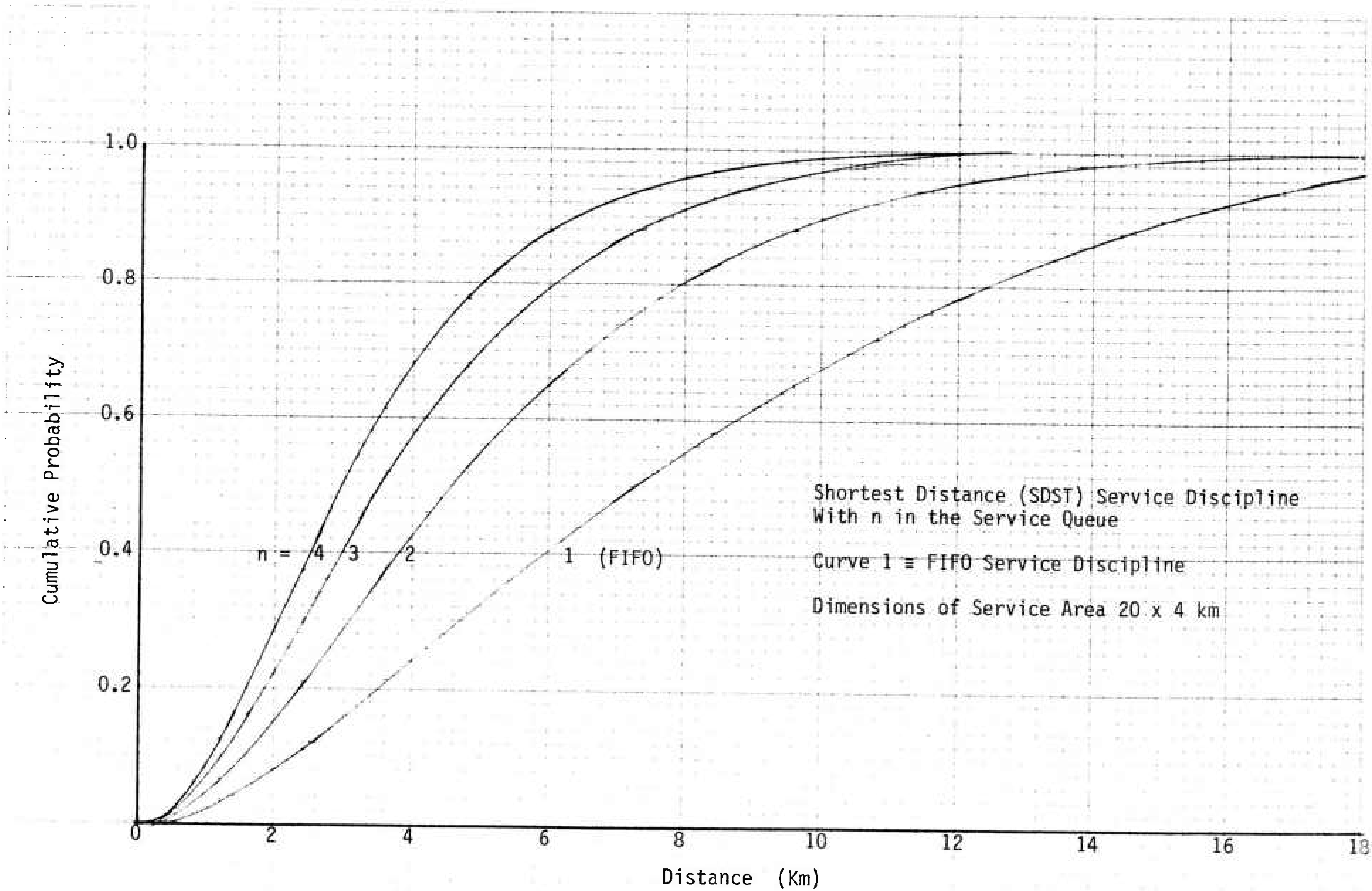

Figure 1. Conditional Cum Probability Distributions of Intercustomer Distance for Several Situations Under SDST Service Discipline

be G(n,d) and the corresponding C.D.F. using FIFO be F(d), for intercustomer distance d. Then, these probability distributions are related as follows:

$$G(n,d) = 1 - (1 - F(d))^{n} \quad .$$

8. The auxiliary equations yield the mean total time in which a C-team is involved with a customer. This parameter and the mean time between failures of operational units (MTBF) are inputs to the Markov queueing model (Annex B). The outputs of this model are typical of stochastic service models: (a) Average and standard deviation of number of individuals in the service system, i.e., either waiting (in a service queue) for a C-team to be assigned or being "served". In this context "being served" means that a C-team is either in transit or conducting repairs, whereas the expression "in the service system", used in queueing theory, means that an individual operational unit has experienced a failure (of the type which is repairable by a C-team), which has not been repaired. In this context, "in the service system" just means "nonoperational".
(b) Avg and S.D. of number of individuals in the service queue. In this situation there is only one service queue, since failed units request service from a single source, the dispatcher.
(c) Avg and S.D. of number of busy service channels (or busy C-teams).
(d) Avg and S.D. of waiting time in the service system, i.e., the time a customer is "down" with a failure, repairable by a C-team.
(e) Avg and S.D. of waiting time in the service queue. This time interval starts with a request for service and ends when a C-team is committed to serve the requesting unit.

9. Model runs were made under two contingencies (or options) for a battalion population of 36 operational units. In the first option the 36 units are divided into batteries of 12 units, each of which has a single C-team dedicated to serving that battery. In the second option any of the 3 C-teams can serve any customer. The latter option does not "fence off" requests by the artifice of restricting the population which a C-team can serve.

10. Using a (battery) population of 12 units, the effect of intercustomer speed on maintenance performance is shown in Table 2. The average speed is increased from 10 to 40 km/hr, in the first three runs -- columns 1 thru 3. In the fourth run the range of speed is increased while preserving the average at the nominal value, 20 km/hr. This may be compared with the results in column 2 to determine the effect of variability in the speed. In the fifth run, the density of C-teams is tripled to shown density effects.

TABLE 2

PARAMETRIC EFFECTS OF CONTACT-TEAM MOBILITY AND DENSITY ON SERVICE

Attribute of the Performance	Service System Parameters: Teams per Btry, Range of Speed				
	1, 10+-5	1, 20+-5	1, 40+-5	1, 20+-10	3, 20+-5 km/hr
Avg Num in Sys*	0.648	0.546	0.503	0.552	0.399
Avg Units in Q	0.209	0.156	0.153	0.159	0.001
Avg Wait in Sys	5.712	4.770	4.375	4.819	3.439 Hrs
Avg Wait in Q	1.845	1.362	1.174	1.385	0.005 Hrs
Pr(Q Wait[8 hr)	0.933	0.961	0.970	0.959	0.999

* "Average Customers in the Service System" is equivalent to the number of customer units "down" for service by a contact team.

11. Using the second option, a series of runs were made with MTBF and MTTR as parameters. The excusrions in these parameters were chosen to develop the locus of points, in the space of these parameters, on which the average number of nonoperational customer units has the constant value -- 0.5, 1.0, 1.5, or 2.0. This family of curves is displayed in Figure 2. These curves are isoavailability curves having availabilities of 98.6%, 97.2%, 95.8%, and 94.4% for the supported system, respectively. An alternative way of displaying these data is shown in Figure 3. In this figure the average number of customer units "down" is shown as a function of MTBF, with MTTR as a parameter. Note that the curve for each particular MTTR shows a "knee" at which the rate of change of average nonoperational units changes remarkably.

TABLE 3

SERVICE PERFORMANCE FOR A SYSTEM OF 3 C-TEAMS SERVING 36 CUSTOMERS

MTBF (Hrs)	Avg Customers in Service Sys and Avg Wait (Hrs) in Service Sys for an MTTR (Hrs): 2		3		4		5	
40	2.558	3.059						
60	1.496	2.602	2.342	4.175				
75			1.736	3.800	2.465	5.512		
80	1.088	2.494						
100	0.862	2.454	1.244	3.580	1.672	4.872	2.183	6.454
120	0.716	2.436						
125			0.981	3.500	1.292	4.653		
150			0.812	3.464	1.061	4.555	1.324	5.729
160	0.536	2.421						
200	0.430	2.415	0.608	3.433	0.788	4.474	0.972	5.551
250			0.486	3.422	0.629	4.444	0.773	5.485
300			0.405	3.416	0.524	4.429	0.643	5.454
350					0.449	4.422	0.551	5.438
400							0.482	5.429

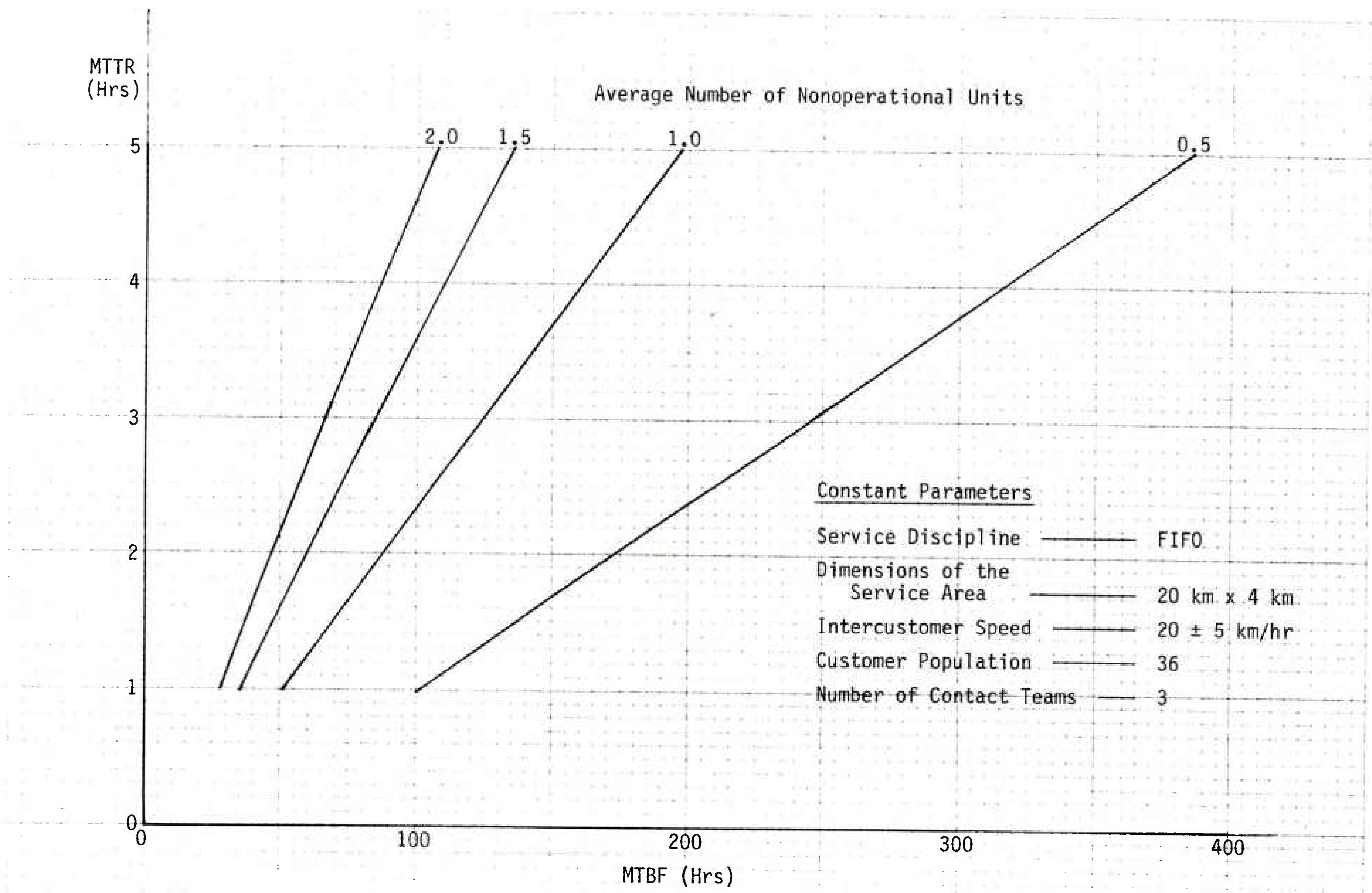

Figure 2. Customer Isoavailability Curves in the Space of MTBF and MTTR

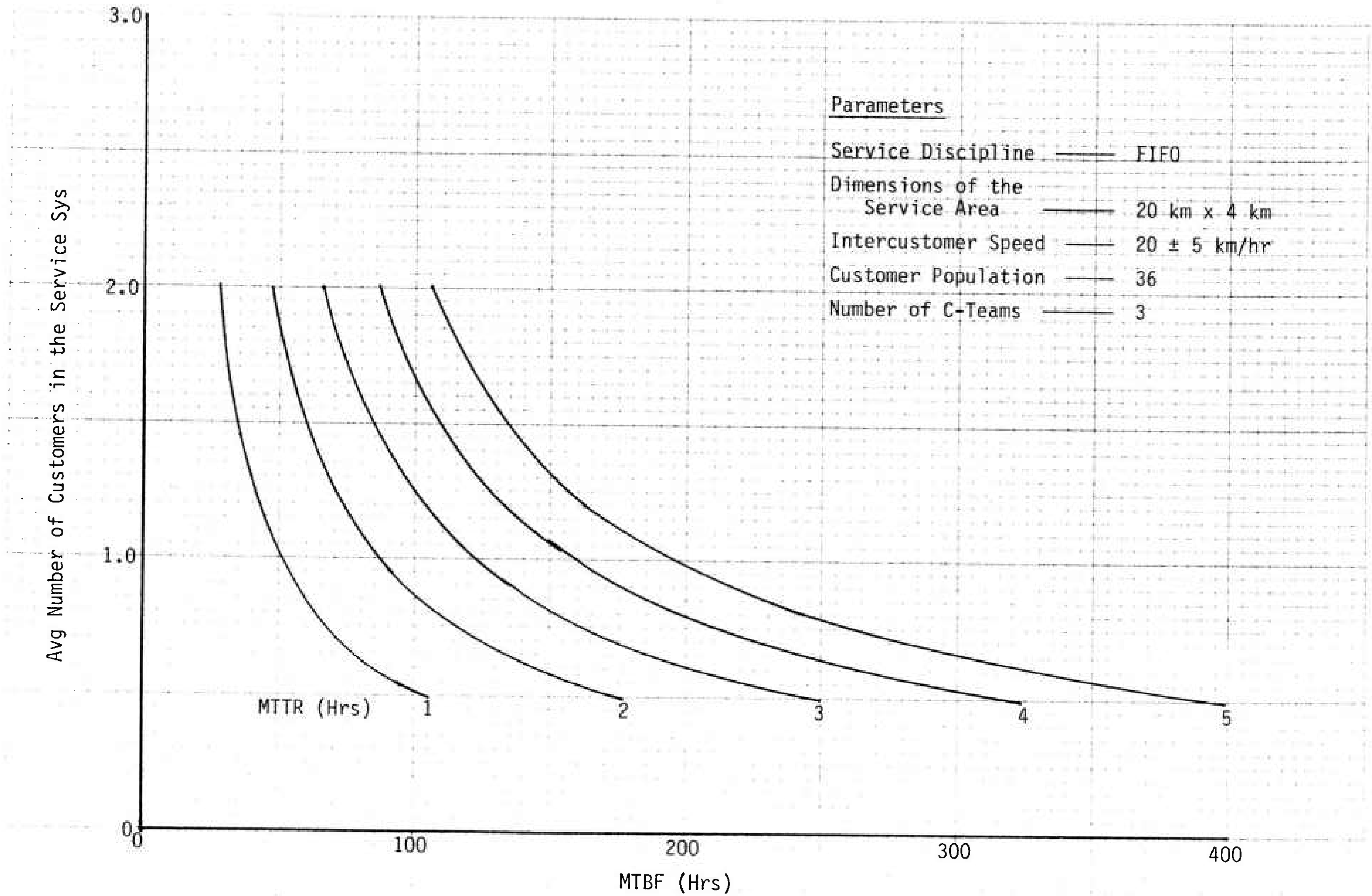

Figure 3. Average Nonoperational Customer Units as a Function of MTBF with MTTR as a Parameter

12. The results presented above were obtained for one statistical model of the time for a C-team to "serve" a customer, i.e., to reach a customer and to diagnose and repair the fault. This model uses a gamma probability distribution, with shape parameter of 2. This is denoted as the gamma(2) model. Among the alternative models one might reasonably take are: (a) the service activity time is exponentially distributed, or (b) the travel time is exponentially distributed (as a 1st stage of service), and the actual repair time is exponentially distributed, as a 2nd stage of service. To examine the sensitivity of the queueing results to the distributional assumption, comparison runs were made for the gamma(2) and exponential models. A gamma(2) model is actually equivalent to two tandem exponential models having identical rate parameters. Therefore, the gamma(2) and exponential models produce results which bound results of model (b), when all models are constrained to have the same mean activity time. The results of the paired comparisons are shown in Table 4. Note that when the C-teams are relatively inactive, the model alternatives produce results which are quite close, particularly with respect to the average number of units in the service system. The values of parameters which are treated as constants in Table 4 are:

Dimensions of the Service Area	20 km X 4 km
Intercustomer Speed	20 +- 5 km/hr
Customer Population Density	36
Number of Supporting C-Teams	3

The two parameters shown in Table 4 are MTBF, the mean time (hours) between service requests per unit, and MTTR, the mean diagnostic and repair time (hours).

TABLE 4

COMPARISON OF QUEUEING STATISTICS FOR ALTERNATIVE DISTRIBUTIONS OF C-TEAM TRAVEL-AND-SERVICE TIME

C.D.F. Total Time	Statistical Attribute of C-Team Performance: Avg No in Sys	S.D. No in Sys	Avg No in Queue	Avg Busy Servers	S.D.Busy Servers	Avg-Sys Wait Tm	Avg-Queue Wait Time
MTBF = 60 MTTR = 1							
Gamma(2)	0.8400	0.9213	0.0147	0.8253	0.8737	1.433	0.025
Exponen	0.8433	0.9347	0.0181	0.8252	0.8748	1.439	0.031
MTBF = 75 MTTR = 1							
Gamma(2)	0.6697	0.8183	0.0063	0.6634	0.7946	1.422	0.013
Exponen	0.6711	0.8247	0.0078	0.6634	0.7953	1.425	0.016
MTBF =100 MTTR = 1							
Gamma(2)	0.5020	0.7066	0.0021	0.4999	0.6971	1.414	0.006
Exponen	0.5025	0.7090	0.0026	0.4999	0.6974	1.416	0.007
MTBF =125 MTTR = 1							
Gamma(2)	0.4020	0.6319	0.0009	0.4011	0.6273	1.411	0.003
Exponen	0.4022	0.6331	0.0011	0.4011	0.6275	1.412	0.004
MTBF = 80 MTTR = 2							
Gamma(2)	1.0885	1.0619	0.0375	1.0510	0.9593	2.494	0.086
Exponen	1.0973	1.0916	0.0466	1.0507	0.9612	2.515	0.107
MTBF =100 MTTR = 2							
Gamma(2)	0.8624	0.9343	0.0162	0.8462	0.8828	2.454	0.046
Exponen	0.8661	0.9489	0.0200	0.8461	0.8840	2.465	0.057
MTBF =100 MTTR = 3							
Gamma(2)	1.2444	1.1474	0.0598	1.1846	0.9977	3.580	0.172
Exponen	1.2588	1.1914	0.0747	1.1841	1.0001	3.623	0.215

13. Conclusions

Several conclusions are derived from the parametric analysis:

(a) Restricting the C-teams so that one team serves only a particular battery has the effect of significantly reducing the readiness of Battalion units. If three teams can serve any of 36 customer units on a dispatched, FIFO basis, the average number of nonoperational units is 1.24, versus 1.63, if the 36 units are divided into 3 exclusive service groups, served by dedicated teams. A comparison of average waiting time in the service area is 3.58 hours, in the first case, versus 4.77 hours, in the second -- a 33 % increase.

(b) Little performance benefit is obtained from increasing the average C-team speed from 20 to 40 km/hr. For such a change, with one team serving 12 customers, the average number of customers down for this sort of maintenance changes from 0.546 to 0.503. The average time spent "in the maintenance system" goes from 4.77 hrs to 4.38 hrs with this speed doubling.

(c) Maintenance performance of the C-teams suffers a serious degradation with a reduction of the average speed from 20 km/hr (nominal) to 10 km/hr. This halving of the speed increases the average number of customers in this service system from 0.546 to 0.648 units (19 % increase). The average time spent in the service system, i.e.,"down", goes from 4.77 hrs to 5.71 hrs, a 20 % increase. These and other parametric effects are displayed in Tables 1, 2, 3, and 4 and Figures 2 and 3.

(d) A knee occurs at an MTBF of 100 hrs in the functional relation of average customers in the service system versus MTBF, for an MTTR of 3 hrs. The location of the knee is somewhat subjective. In this case, a quite rapid increase occurs in the average number of customers down for maintenance with a small decrease in the MTBF below the location of the knee. The location of the knee decreases with decreasing MTTR. Decreasing the MTTR also makes the knee more pronounced.

(e) Several alternatives were examined to describe the probability distribution of the time for a C-team to travel to a customer and to complete maintenance service on that customer. In general, the statistics which characterize service performance are relatively insensitive to the form of this distribution, given a mean value of the total "service" time.

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ANNEX A

DERIVATION OF EQUATIONS

Probability Distribution for Intercustomer Distance

The cumulative distribution function (c.d.f.) for the distance a C-team travels between customers is derived in a series of steps. We start by observing that this distance consists of the sum of two independent random variables, each of which is the projection of intercustomer distance along a line. This fact is a consequence of the requirement that the path followed by a C-team consists of series of orthogonal segments, each of which is parallel to one of the sides of the rectangular service area. One may equivalently view this type of path as an x-displacement (from the customer just served) followed by a y-displacement. Let the dimensions of the service area be: a by b, so that 0 le x le a, 0 le y le b. (Note that "le" denotes "less than or equal to". Similarly, "ge" is the abbreviation for "greater than or equal to", etc.). Suppose that the c.d.f. of the x-component of displacement is denoted by $F_1(d_1)$ and that for the y-component is $F_2(d_2)$, where the intercustomer distance, d, is

$$d = d_1 + d_2, \qquad 0 \text{ le } d \text{ le } a+b. \tag{1}$$

Because the components of d are independent, the c.d.f. of d, $F_d(d)$, is the convolution of F_1 and F_2. The method just outlined is the approach used to obtain the distribution of intercustomer distance, using a FIFO service discipline.

As a first step, consider the distance between two randomly selected points along a line segment of unit length (0,1), where the points are uniformly distributed. Let the random location of the first point be X_1 and that for the second be X_2. Note that upper case letters are used for realizations of a random variable. The random variable denoting the distance is Z:

$$Z = \text{abs}(X_1 - X_2). \tag{2}$$

Equivalently,

$$Z = X_1 - X_2, \text{ for } X_1 \text{ ge } X_2,$$

or

$$Z = X_2 - X_1, \text{ for } X_2 \text{ ge } X_1. \tag{3}$$

Notationally, let the c.d.f. of z be

$$F_z(z) = \Pr(Z \text{ le } z). \tag{4}$$

Because of symmetry, $\Pr(X_1 \text{ ge } X_2)$ is equal to $\Pr(X_2 \text{ ge } X_1)$.

Therefore, consider only the first of these two contingencies, in which $X_1 \ge X_2$.

Notationally, let I_u represent the integration operator over the feasible space of the variable of integration u. To be specific the limits of integration -- u1, u2 -- may be denoted as arguments in this way: $I_u(u1,u2)$. To abbreviate, the expression $(X_i = u)$ means that the random variable X_i lies between u and u+du, (i = 1,2). Then,

$$F_z(z) = I_u \Pr(X_2 = u)\Pr(X_1 - u \le z). \tag{5}$$

In performing the integration, one must choose limits so that $0 \le u \le 1$ and that $X_1 \ge X_2 = u$, as assumed.

From (4) and (5), remembering that $\Pr(X_i = u) = du$,

$$F_z(z) = I_u \; du \Pr(u \le X_1 \le z+u)$$

$$F_z(z) = I_u(0,1-z) \; du \; z \; + \; \Pr(u \le z). \tag{6}$$

After integrating, evaluating at the limits, and combining terms,

$$F_z(z) = 2z - z^2. \tag{7}$$

By a change of variable with $z = d_1/a$,

$$F_1(d_1) = 2(d_1/a) - (d_1/a)^2, \qquad d_1 \le a. \tag{8}$$

The probability density function (p.d.f.) for d_1 can be obtained from (8) by differentiation, yielding

$$f_1(d_1) = (2/a)(1 - d_1/a), \qquad d_1 \le a. \tag{9}$$

Taking the first two origin moments of f_1 -- $E(d_1)$ and $E(d_1^2)$, where

$$E(d_1^k) = I_u(0,a) \; u^k f_1(u) \; du, \qquad k = 1,2, \tag{10}$$

$$E(d_1) = a/3 \tag{11a}$$

and

$$E(d_1^2) = a^2/6 \ . \tag{11b}$$

The variance of d_1, $V(d_1)$, is obtained from (11) via the difference between second origin moment and mean squared:

$$V(d_1) = a^2/18. \tag{12}$$

The c.d.f., p.d.f., and associated moments of d_2 can be developed using the above results for d_1 by merely substituting b for a in equations (8) thru (12). Note that the random variable for the x-component of d differs statistically from that of the y-component only in its range. Thus,

$$F_2(d_2) = 2(d_2/b) - (d_2/b)^2 , \qquad d_2 \text{ le } b. \tag{13}$$

$$f_2(d_2) = (2/b)(1 - d_2/b) , \qquad d_2 \text{ le } b. \tag{14}$$

The mean and variance of f_2 are

$$E(d_2) = b/3 \tag{15a}$$

and

$$V(d_2) = b^2/18 \ . \tag{15b}$$

Since the mean of the sum of independent random variables is just the sum of the means, from (1), (11a), and (15a),

$$E(d) = (a + b)/3. \tag{16}$$

The same additive law applies to the variance of the sum of independent random variables. Thus,

$$V(d) = (a^2 + b^2)/18. \tag{17}$$

These results also appear on page 4 of this report. By contrast to the other equations on page 4, these results are exact.

Notationally, let the p.d.f. of d be denoted by $f_d(d)$.
As noted above, the probability distribution of d is the convolution of the distributions of its components. This statement also applies to the densities. Both $f_1(h)$ and $f_2(v)$ are defined on finite domains: (0,a) and (0,b), respectively. Therefore, the domain of $f_d(d)$, with $d = h+v$, is (0,a+b).
It will be seen that the functional form changes over subdomains such as (0,min(a,b)) and (min(a,b),max(a,b)). This results from the finite limits of integration in the convolution operation. The specification of these limits in the space of h and v is tricky. A straightforward way of handling this problem is to use Laplace transforms. The convolution theorem of integral transforms applies here: the transform of the convolution is the product of the transforms of the functions being convolved. Denoting the Laplace transform of f_i as $f^*_i(s)$, where i is 1 or 2,

$$f^*_i(s) = I_u\ (0,liu(i))\ \exp(-su)\ f_i(u)\ du, \qquad (18)$$

with liu(1) = a and liu(2) = b.
Then,

$$f^*_d(s) = f^*_1(s)\ f^*_2(s). \qquad (19)$$

In performing the inverse transform, the "problem" of subdomains is handled automtically by the mechanics of the operation. This attractive aspect of integral transforms motivated this approach.

After some algebraic manipulation of the results from (18), the transform for f_1 becomes

$$f^*_1(s) = (2/a)/s - (2/a^2)/s^2 + (2/a^2)\exp(-sa)/s^2. \qquad (20)$$

A comparable expression for $f^*_2(s)$ is obtained by substituting b for a in equation (20). Note that terms involving exp(-sa) and exp(-sb) appear in $f^*_1(s)$ and $f^*_2(s)$, respectively.
Using (19), one notes that terms involving these factors and the factor exp(-s(a+b)) appear in $f^*_d(s)$. During inversion, terms which involve a factor of the form exp(-cs), with c constant, are converted to terms in the inverse which have a unit step function as a factor. The unit step function in the variable d is defined as

$$\begin{aligned} u(d - c) &= 0, & d \text{ lt } c \\ &= 1/2, & d = c \\ &= 1, & d \text{ gt } c. \end{aligned} \qquad (21)$$

Thus, terms in the inverse which have a unit step factor contribute only to the result for that portion of the domain for which d is greater than or equal to the parameter c.

From (19) and (20),

$$f^*_d(s) = (1/s^2)A + (1/s^3)(B + B_a \exp(-sa) + B_b \exp(-sb)) +$$

$$(1/s^4)(C - C \exp(-sa) - C \exp(-sb) + C \exp(-s(a+b))) ,$$

with

$$A = 4/(ab),$$

$$B = -4(a + b)/(ab)^2 , B_a = 4/a^2 b \text{ and } B_b = 4/ab^2 ,$$

$$C = 4/(ab)^2 . \qquad (22)$$

Terms in this transform involving exp(-s(a+b)) are needed to insure that the inverse transform (p.d.f.) is zero for d ge a+b. By restricting the domain of the p.d.f. to (0, a+b), these terms can be neglected. With this restriction, the inverse transform becomes.

$$f_d(d) = Ad + (B/2)d^2 + (B_a/2)(d-a)^2 u(d-a) + (B_b/2)(d-b)^2 u(d-b)$$

$$+ (C/6)(d^3 - (d-a)^3 u(d-a) - (d-b)^3 u(d-b)), \text{ d le a+b.} \qquad (23)$$

From (21) and (23), it can be seen that the polynomial in d for the p.d.f. assumes different forms over the subdomains: (0, min(a,b)), (min(a,b),max(a,b)), and (max(a,b),a+b). The c.d.f. of intercustomer distance is obtained from (23) by integration:

$$F_d(d) = I_t(0,d) f_d(t) dt, \qquad \text{d le a+b.} \qquad (24)$$

The implementing computer code for the result of (24) is in Annex B on page B-7.

The c.d.f. of intercustomer distance from (24) was derived assuming the coordinates of the positions of the two customers were statistically independent and uniformly distributed. This situation exists under both FIFO service discipline and when less than 2 customers are queued with SDST discipline. Suppose the service discipline is shortest distance (SDST). Denote the conditional c.d.f. of d, given that n customers are waiting, by G(n,d). By definition,

$$\Pr(D \text{ gt } d, n) = 1 - G(n,d). \qquad (25)$$

But,

$$\Pr(D \text{ gt } d, n) = \Pr(D_1 \text{ gt } d)\Pr(D_2 \text{ gt } d) \ldots \Pr(D_n \text{ gt } d),$$

where D_n is the random value of the n th distance from $F_d(d)$.

Thus,

$$Pr(D \text{ gt } d,\ n) = (1 - F_d(d))^n ,$$

and

$$G(n,d) = 1 - (1 - F_d(d))^n . \qquad (26)$$

The unconditional c.d.f. of d under SDST discipline, G(d), is obtained by summing, over n, products of the form:

Pr(number queued = n)G(d,n).

The probability weights in the resulting expression can be obtained from the analytic queueing model. An iterative evaluation of these probabilities is required, since the service rate, being a model input, depends upon the mean and variance of d. The first two origin moments of G(d) can be obtained by numerical integration of the following equations using Simpson's rule.

$$E(d) = I_t\ (0,d)(1 - G(t))\ dt \qquad (27a)$$

$$E(d^2) = I_t\ (0,d)\ t\ (1 - G(t))\ dt. \qquad (27b)$$

The computer program which implements these operations for any c.d.f., G(d), is found in Annex B, starting on page B-24.

Approximations for Mean and Variance of Travel Time

Approximations are given on page 4 for the mean and variance of intercustomer travel time. The derivation of these results is sketched here. The intercustomer time, t, is the ratio of the distance, d, to the (average) speed, s, over that distance. This (avg) speed is considered a random variable, varying from one customer to the next. The mean and variance of s are considered as given. Since t is a function of d and s, it can be expanded in a Maclaurin series about the mean values of d and of s. The mathematical expectation of both sides of this equation is taken. Cubic- and higher -terms of d - E(d) and of s - E(s) are discarded. This procedure leads to the approximation for E(t), given on p. 4. The Maclaurin expansion for t is squared, and mathematical expectations are taken on both sides of this equation. After discarding the higher-order terms, one obtains an approximation for the second origin moment of t. The approximation for the variance of t is this second origin moment minus the square of the above mean. The neglect of the 3rd and higher central moments in the above derivations may cause one to be concerned about the accuracy of these approximations when the distributions of d and of s are rather skewed. To evaluate the error of approximation one can compare these results with exact results for a special case. The special case, discussed next, involves distributional assumptions for intercustomer distance and speed. The c.d.f. of t for the special case is given in closed form. Using this expression, the mean and variance of t are calculated via the procedure in (27). The aproximation has nearly 3-digit accuracy for an example with a = 15 km and with s uniform over the interval 15 to 25 km/hr.

Derivation of the Distribution of Travel Time for a Special Case

To obtain a probability distribution of intercustomer distance, let the population be distributed uniformly along the x-axis, from zero to a. In this case the FIFO c.d.f. of intercustomer distance is given by (8), with the p.d.f. given by (9). Note that the distance distribution is positively skewed, and, thus has a non-zero 3rd central moment -- contrary to the above implicit assumption. For the present purpose, denote the p.d.f. and c.d.f. of the intercustomer distance, x, as $f_x(x)$ and $F_x(x)$, respectively.

To derive the c.d.f. of the intercustomer travel time, assume that the speed has a uniform distribution between u and v. Thus,

$$f_s(s) = 1/(v-u), \qquad u \le s \le v. \tag{28}$$

Notationally, let the c.d.f. of time, t, be $F_t(t)$, where

$$F_t(t) = \Pr(T \le t), \text{ for a random time } T. \tag{29}$$

Observe that T can take any value from zero to (a/u). For a particular realization, $T = X/S$, with X being the particular distance and S being the particular value of the intercustomer speed. Then, the conditions for $T \le t$ are :

$X = x$ and $v \ge S \ge x/t$, with $0 \le x \le a$.

Applying these conditions to (29),

$$F_t(t) = \int f_x(x) \Pr(S \ge x/t)\, dx, \tag{30a}$$

where

$$\begin{aligned} \Pr(S \ge x/t) &= 1, && x \le ut, \\ &= 1 - (x/t - u)/(v-u), && u \le x/t \le v, \\ &= 0, && x > vt. \end{aligned} \tag{30b}$$

Thus,

$$F_t(t) = \int_{0}^{ut} f_x(x)\, dx + \int_{ut}^{a} f_x(x)(1 - (x/t-u)/(v-u))\, dx,$$

for $vt > a$, (31a)

$$F_t(t) = F_x(ut) + \int_{ut}^{vt} f_x(x)(1 - (x/t-u)/(v-u))\, dx,$$

for $vt \le a$, or for $t \le a/v$. (31b)

Carrying out the indicated integrations yields the following results. For $va/u > vt > a$, (31a) becomes

$$\begin{aligned} F_t(t) = {} & 1 - a/(3t(v-u)) + u^2 t/(a(v-u)) - (2/3)u^3 t^2/(a^2 (v-u)) \\ & + (u/(v-u))(1 - F_x(ut)), \qquad a/u \ge t > a/v. \end{aligned} \tag{32a}$$

$$\begin{aligned} F_t(t) = {} & F_x(vt)(1 + u/(v-u)) - F_x(ut)\, u/(v-u) \\ & - (v+u)/a^2\, t + 2/(3a^3 (v-u))(v^3 - u^3)t^2, \qquad t \le a/v. \end{aligned} \tag{32b}$$

ANNEX B

COMPUTER SOURCE PROGRAMS

The source programs listed here are written in SIMSCRIPT 2.5 for the PR1ME 750 minicomputer. However, the code does not employ features unique to this computer. The MAIN and subprograms calculate statistics for the stochastic steady state of a maintenance service system. This system consists of several mobile contact teams (C-teams) which are centrally dispatched. The teams serve a fixed population of customers within a service area. The customers are assumed to be uniformly distributed within a rectangular service area whose dimensions are input parameters. All program inputs are read interactively, with prompting messages displayed at the terminal. At the beginning of each program listing is found: (a) a functional description of the program, (b) a list of the program inputs -- or input arguments, for subprograms, (c) a list of the program outputs, and, in some instances, (d) a list of key endogenous variables. All utility functions and routines are included in this listing. No external files are used. Since the output is lengthy, it is necessary to set up a COMO file to display all of it and to obtain a permanent copy.

The programs found in this annex are located as follows:

```
PREAMBLE ''CONTACT.Q
NORMALLY MODE IS REAL
DEFINE PDT.FIFO AS A REAL FUNCTION WITH 3 ARGUMENTS
DEFINE CDT.FIFO AS A REAL FUNCTION WITH 3 ARGUMENTS
DEFINE PDT.SDST AS A REAL FUNCTION WITH 3 ARGUMENTS
DEFINE CDT.SDST AS A REAL FUNCTION WITH 3 ARGUMENTS
DEFINE FUN.CDF TO MEAN CDT.FIFO
END ''PREAMBLE

MAIN ''CONTACT.Q
''
''PROGRAM SOLVES THE STEADY-STATE QUEUEING PROBLEM ASSOCIATED WITH A
''MOBILE CONTACT TEAM WHICH VISITS AND SERVES CUSTOMERS IN A RECTANGULAR
''SERVICE AREA.  THE CUSTOMERS, REQUIRING MAINT SERVICE, ARE ASSUMED
''TO BE UNIFORMLY DISTRIBUTED WITHIN A RECTANGLE WITH DIMENSIONS ARANGE
''BY BRANGE.  THE POPULATION OF CUSTOMERS IS FINITE (= MPOP).  THE NUMBER
''OF C-TEAMS IS EQUIVALENT TO THE NUMBER OF SERVICE CHANNELS (NSERVE).
''TWO TYPES OF SERVICE DISCIPLINE ARE PERMITTED:  FIFO AND SDST.  USING
''FIFO, THE CONTACT TEAM TRAVELS TO THE CUSTOMER HAVING THE FIRST REQUEST.
''USING SDST, THE CONTACT TEAM TRAVELS TO THE CUSTOMER HAVING THE SHORTEST
''DISTANCE FROM THE CURRENT LOCATION.  AN ERLANG(2) DISTRIBUTION IS USED
''TO APPROXIMATE THE TOTAL TIME TO TRAVEL TO AND SERVE A CUSTOMER.
''
''INPUTS:
''FLAG.DIST ___ AN INTEGER FLAG TO INDICATE PRINTING OF THE PROBABILITY
''              DISTRIBUTION OF INTERCUSTOMER DISTANCE.
''FLAG.FIFO ___ AN INTEGER FLAG TO INDICATE A FIFO SERVICE DISCIPLINE
''              (= 1).  THE ALTERNATIVE DISCIPLINE IS TO SERVE THE CUST-
''              OMER HAVING THE SHORTEST DISTANCE (SDST) NEXT.
''IPRINT ______ FLAG TO PRINT QUEUEING STATISTICS FROM A SUBROUTINE.
''ARANGE ______ THE FRONTAL DIMENSION (KM) OF THE RECTANGULAR CUSTOMER
''              TERRITORY SERVED BY A SET OF CONTACT TEAMS.
''BRANGE ______ THE CROSS-FRONTAL (ORTHOGONAL) DIMENSION (KM) OF THE
''              RECTANGULAR CUSTOMER TERRITORY.
''SPEED _______ THE AVERAGE SPEED (KM/HR) OF MOVEMENT OF A CONTACT TEAM
''              BETWEEN CUSTOMERS.
''MPOP ________ SIZE OF THE CUSTOMER POPULATION, I.E., THE NUMBER OF UNITS
''              SERVED BY CONTACT TEAMS.
''NSERVE ______ THE NUMBER OF CONTACT TEAMS (OR SERVICE CHANNELS).
''MTBF ________ MEAN TIME (HOURS) BETWEEN FAILURES OF THE AGGEGATE OF PARTS
''              REQUIRING SERVICE BY A CONTACT TEAM.
''MTTR ________ MEAN TIME TO DIAGNOSE AND REPAIR A FAILURE WHICH REQUIRES
''              A CONTACT-TEAM SERVICE REQUEST.
''
''ENDOGENOUS VARIABLES:
''P.NSTATE(*) _ A REAL VECTOR OF MARKOV-STATE PROBABILITIES FOR THE SERVICE
''              SYSTEM.  P.NSTATE(N) IS THE PROB THAT N UNITS ARE IN THE
''              "SERVICE SYSTEM", I.E., EITHER BEING SERVED OR AWAITING
''              SERVICE.
''LAMBDA ______ AVERAGE ARRIVAL RATE PER CUSTOMER (PER HOUR).
''MU1 _________ AVERAGE UNIT SERVICE RATE, INCLUDING TRAVEL, (PER HOUR).
''MLIM ________ LIMITING NUMBER OF CUSTOMER PERMITTED IN THE SERVICE SYS.
''PLIM ________ PROB ACCURACY IN C.D.F., TO LIMIT THE MARKOV STATE SPACE.
```

```
''
''OUTPUTS:
''P.NULL ______ STEADY-STATE PROBABILITY THAT THE SERVICE SYSTEM IS EMPTY.
''P.SYS.FULL __ PROBABILITY THAT ALL SERVICE CHANNELS ARE BUSY.
''P.CUST.WAIT _ PROB THAT AN "ARRIVING" CUSTOMER, I.E., A JUST FAILED
''              UNIT, MUST QUEUE.
''ENSYS _______ EXPECTED NUMBER OF CUSTOMER UNITS DOWN WITH A FAILURE,
''              WHICH REQUIRES THE USE OF A CONTACT TEAM.
''SDNSYS ______ STANDARD DEVIATION OF CUSTOMER UNITS IN THE SERVICE SYS.
''ENQ _________ EXPECTED NUMBER OF CUSTOMER UNITS WAITING FOR A SERVICE
''              CHANNEL.
''ESYS.WAIT ___ EXPECTED TIME THAT A FAILED CUSTOMER UNIT MUST WAIT UNTIL
''              SERVICE BY A CONTACT TEAM IS COMPLETE.
''EQ.WAIT _____ EXPECTED WAITING TIME IN THE SERVICE QUEUE.
''EQ.WAIT.GQ __ CONDITIONAL AVERAGE WAITING TIME IN THE SERVICE QUEUE,
''              GIVEN THAT THE CUSTOMER MUST QUEUE.
''SDQ.WAIT.GQ _ CONDITIONAL STANDARD DEVIATION OF THE WAITING TIME IN THE
''              SERVICE QUEUE, GIVEN THAT THE CUSTOMER MUST QUEUE.
''E.BUSY.SERVERS_AVERAGE NUMBER OF CONTACT TEAMS WHICH ARE BUSY, I.E.,
''              THAT ARE DISPATCHED TO A CUSTOMER OR PERFORMING SERVICE.
''SD.BUSY.SERVERS_STANDARD DEVIATION OF THE NUMBER OF BUSY SERVERS (TEAMS).
''
DEFINE FLAG.DIST,FLAG.FIFO,I,IPRINT,J,K,L,M,MLIM,MPOP,N,NSERVE AS INTEGER
VARIABLES
DEFINE ANSWER AS A TEXT VARIABLE
DEFINE P.NSTATE AS A REAL, 1-DIMENSIONAL ARRAY
     PRINT 1 LINE THUS
DO YOU WANT FIFO SERVICE?  (Y OR N).  ALTERNATIVE IS SHORTEST DISTANCE.
     READ ANSWER
     IF SUBSTR.F(ANSWER,1,1) = "Y"
         LET FLAG.FIFO=1
     OTHERWISE
         LET FLAG.FIFO=0
     ALWAYS
     PRINT 1 LINE THUS
INPUT THE FRONTAL DIMENSION (KM) OF THE AREA OF THE CONTACT TEAM(S).
     READ ARANGE
     PRINT 1 LINE THUS
INPUT THE DEPTH (CROSS-FRONTAL) DIMENSION (KM) OF THE AREA OF THE TEAM(S).
     READ BRANGE
     PRINT 1 LINE THUS
INPUT THE AVG SPEED (KM/HR) OF THE CONTACT TEAM BETWEEN CUSTOMERS.
     READ SPEED
     PRINT 1 LINE THUS
INPUT THE PLUS OR MINUS MAX DEVIATION IN SPEED FROM AVERAGE (KM/HR).
     READ DELTAS
     LET DELTAS=MIN.F(SPEED/2.0,DELTAS)
     LET VS=DELTAS**2/3.0
     PRINT 1 LINE THUS
INPUT THE NUMBER OF CUSTOMERS (FIRE UNITS) SERVED BY THE CONTACT TEAMS.
     READ MPOP
     RESERVE P.NSTATE(*) AS MPOP
     PRINT 1 LINE THUS
INPUT THE NUMBER OF CONTACT TEAMS (SERVICE CHANNELS) CONSIDERED, LE 3.
     READ NSERVE
     LET NSERVE=MIN.F(3,NSERVE)
```

```
        PRINT 1 LINE THUS
  INPUT THE MEAN TIME (HRS) BETWEEN PERTINENT FAILURES OF A FIRE UNIT.
        READ MTBF
        PRINT 1 LINE THUS
  INPUT THE AVERAGE DIAGNOSTIC AND REPAIR TIME (HRS) FOR THESE FAILURES.
        READ MTTR
        PRINT 1 LINE THUS
  DO YOU WANT TO PRINT THE PROB DIST OF INTERCUSTOMER DISTANCE?  (Y OR N).
        READ ANSWER
        IF SUBSTR.F(ANSWER,1,1) = "Y"
            LET FLAG.DIST=1
        OTHERWISE
            LET FLAG.DIST=2
        ALWAYS
        PRINT 1 LINE THUS
  DO YOU WANT TO PRINT FROM THE QUEUEING SUBROUTINE?  (Y OR N).
        READ ANSWER
        IF SUBSTR.F(ANSWER,1,1) = "Y"
            LET IPRINT=1
        OTHERWISE
            LET IPRINT=0
        ALWAYS
        SKIP 3 LINES
        PRINT 2 LINES THUS
  INPUT DATA FOR PERFORMANCE OF CONTACT TEAMS

  ''
  ''CALCULATE AVG AND STD DEV OF INTERCUSTOMER DISTANCE AND TRAVEL TIME.
  ''
        IF FLAG.FIFO=1
            PRINT 1 LINE THUS
  SERVICE DISCIPLINE USED IS "FIFO".
            LET ER=ARANGE/3.0+BRANGE/3.0
            LET VR=(ARANGE**2+BRANGE**2)/18.0
        OTHERWISE
            PRINT 1 LINE THUS
  SERVICE DISCIPLINE USED IS "SHORTEST-DISTANCE".
  ''        LET ER=(ARANGE+BRANGE)/5.0
  ''        LET VR=(ARANGE**2+BRANGE**2)*(1.0/15.0-1.0/25.0)
            CALL MINRV (0,2,ARANGE+BRANGE,ARANGE,BRANGE) YIELDING ER,VR
        ALWAYS
        LET ETM.TRVL=ER/SPEED+ER*VS/SPEED**3 ''APPROXIMATELY
        LET VTM.TRVL=VR/SPEED**2+ER**2*VS/SPEED**4+ER**2*VS**2/SPEED**6
        +VR*VS/SPEED**4 ''APPROXIMATELY FROM MACLAUREN EXPANSION ABOUT AVG'S
        PRINT 7 LINES WITH ARANGE,BRANGE,SPEED,DELTAS,MPOP,NSERVE,MTBF,MTTR
        THUS
  FRONTAL DIMENSION (KM) OF SERVICE AREA ___ **.****
  DEPTH DIMENSION (KM) OF SERVICE AREA _____ **.****
  RANGE OF SPEED (KM/HR) OF CONTACT TEAM ___ **.**** +/- **.****
  NUMBER OF CUST UNITS IN SERVICE AREA _____ **
  NUMBER OF CONTACT TEAMS IN SERVICE AREA __ **
  AVG TIME (HR) BET SERVICE REQUESTS/CUST __ ****.**
  AVERAGE DIAGNOSTIC AND REPAIR TIME (HR) __ **.****
        SKIP 1 LINE
```

```
      PRINT 7 LINES WITH ER,SQRT.F(VR),SPEED,SQRT.F(VS),ETM.TRVL,
      SQRT.F(VTM.TRVL),ETM.TRVL/(ETM.TRVL+MTTR)
      THUS
AVERAGE DISTANCE (KM) BETWEEN CUSTOMERS ___ **.****
STD DEV DISTANCE (KM) BETWEEN CUSTOMERS ___ **.****
AVERAGE SPEED (KM/HR) BETWEEN CUSTOMERS ___ **.****
STD DEV SPEED (KM/HR) BETWEEN CUSTOMERS ___ **.****
AVG   TRAVEL TIME (HR) BETWEEN CUSTOMERS ___ **.****
S.D. TRAVEL TIME (HR) BETWEEN CUSTOMERS ___ **.****
RATIO:  TRAVEL TIME/(TRAVEL+REPAIR TIMES) _  *.****
      SKIP 2 LINES
      IF FLAG.DIST NE 1
          GO TO LO
      OTHERWISE
      PRINT 6 LINES THUS
PROBABILITY DISTRIBUTION OF INTERCUSTOMER DISTANCE

------------------------------------------------------------------------
DIST    C.D.F  OF MIN DIST FOR N CUST.  N:
(KM)    FIFO      2         3         4

------------------------------------------------------------------------
      LET DR=(ARANGE+BRANGE)/20.0
      FOR I=1 TO 20 DO
          LET R=DR*I
          LET CDF1=CDT.FIFO(ARANGE,BRANGE,R)
          LET CDF2=1.0-(1.0-CDF1)**2
          LET CDF3=1.0-(1.0-CDF1)**3
          LET CDF4=1.0-(1.0-CDF1)**4
          PRINT 1 LINE WITH R,CDF1,CDF2,CDF3,CDF4
          THUS
**.*** *.****   *.****   *.****   *.****
      IF CDF1 GE 0.9999
          GO TO KO
      OTHERWISE
      LOOP ''OVER I
'KO'PRINT 2 LINES THUS
------------------------------------------------------------------------

'LO'LET LAMBDA=1.0/MTBF
      LET MU1=1.0/(ETM.TRVL+MTTR)
      SKIP 2 LINES
      PRINT 8 LINES WITH LAMBDA,MPOP*LAMBDA,MU1,NSERVE*MU1,MPOP,NSERVE
      THUS
INPUT PARAMETERS FOR A STEADY-STATE, FINITE QUEUEING SYSTEM

ARRIVAL RATE (LAMBDA) ___ **.***** PER HR PER OPERATING CUSTOMER UNIT
MAX ARRIVAL RATE ________ **.***** UNITS PER HR
SERVICE RATE (MU1) ______ **.***** PER HR PER CUSTOMER
MAX SERVICE RATE ________ **.***** UNITS PER HR
CUSTOMER POPULATION _____ **       CUSTOMERS
SERVICE CHANNELS ________ **       SERVERS
```

```
''
''GET A LIMIT ON THE SYSTEM-STATE INDEX (MLIM) WHICH SATISFIES ACCURACY
''REQUIREMENTS.
''
     LET PLIM=0.0001
     CALL LIMSTATE (LAMBDA,MU1,MPOP,NSERVE,PLIM,0) YIELDING MLIM,P.NULL,
     P.NSTATE(*)
     PRINT 1 LINE WITH MLIM,PLIM
     THUS
SYSTEM INDEX = ** FOR ___  *.***** ACCURACY IN CUM PROB
     SKIP 4 LINES
     CALL FINITE.ME2.Q(LAMBDA,MU1,MPOP,MLIM,NSERVE,IPRINT) YIELDING P.NULL,
     P.SYS.FULL,P.CUST.WAIT,ENSYS,SDNSYS,ENQ,ESYS.WAIT,EQ.WAIT,EQ.WAIT.GQ,
     SDQ.WAIT.GQ,E.BUSY.SERVERS,SD.BUSY.SERVERS,P.NSTATE(*)
     SKIP 2 LINES
     IF IPRINT NE 1
          PRINT 7 LINES WITH P.NULL,P.SYS.FULL,ENSYS,SDNSYS,ESYS.WAIT,
          E.BUSY.SERVERS,SD.BUSY.SERVERS
          THUS
PROBABILITY THAT SERVICE SYSTEM IS EMPTY (NO UNITS DOWN) _ *.****
PROBABILITY THAT ALL SERVICE CHANNELS ARE BUSY __________ *.****
AVERAGE NUMBER OF CUSTOMERS  IN THE SERVICE SYSTEM _______ **.***
STD DEV NUMBER OF CUSTOMERS  IN THE SERVICE SYSTEM _______ **.***
UNCOND AVG WAITING TIME (HR) IN THE SERVICE SYSTEM _______ **.***
AVERAGE NUMBER OF BUSY SERVICE CHANNELS __________________ **.***
STD DEV NUMBER OF BUSY SERVICE CHANNELS __________________ **.***
     ALWAYS
     SKIP 2 LINES
     STOP
END ''CONTACT.Q
```

```
FUNCTION CDT.FIFO (A,B,T)
''
''FUNCTION EVALUATES THE CUM DISTRIBUTION FUNCTION (C.D.F) FOR THE
''INTERCUSTOMER DISTANCE OF A DISPACHED MOBILE SERVICE SYSTEM, UNDER A
''FIFO SERVICE DISCIPLINE.  THE CUSTOMER AREA (OR TERRITORY) IS A RECTANGLE
''WITH DIMENSIONS A BY B.  CUSTOMERS ARE ASSUMED TO BE UNIFORMLY DISTRI-
''BUTED IN THIS AREA.  INDEPENDENCE OF THE X- AND Y- COORDINATE POSITIONS
''OF THE CUSTOMERS IS ASSUMED.  THE PATH THAT THE SERVICE SYSTEM IS
''ASSUMED TO FOLLOW IS A SERIES OF ORTHOGONAL SEGMENTS PARALLEL TO THE
''SIDES OF THE RECTANGULAR SERVICE AREA.
''
    IF T  GT  A+B
        RETURN WITH 1.0
    OTHERWISE
    LET AB=A*B
    LET ABS=AB**2
    LET CDF=2.0*T**2/AB*(1.0-T*(A+B)/AB/3.0)+T**4/ABS/6.0
    IF T  GT  B
        ADD (T-B)**3/6.0/AB*(4.0/B-(T-B)/AB) TO CDF
    ALWAYS
    IF T  GT  A
        ADD (T-A)**3/6.0/AB*(4.0/A-(T-A)/AB) TO CDF
    ALWAYS
    RETURN WITH CDF
END ''CDT.FIFO
```

```
ROUTINE FINITE.ME2.Q (LAMBDA,MU1,MPOP,MLIM,NSERVE,MODE) YIELDING P.NULL,
    P.SYS.FULL, P.CUST.WAIT, ENSYS, SDNSYS, ENQ, ESYS.WAIT, EQ.WAIT,
    EQ.WAIT.GQ, SDQ.WAIT.GQ, E.BUSY.SERVERS, SD.BUSY.SERVERS, P.NSTATE
''
''THIS ROUTINE CALCULATES THE STEADY-STATE, SYSTEM STATE-PROBABILITY
''VECTOR FOR A SERVICE SYSTEM HAVING EXPONENTIAL INTERARRIVAL TIMES FOR
''EACH MEMBER OF A FINITE CUSTOMER POPULATION, AND HAVING NSERVE SERVERS,
''WITH SERVICE TIMES DISTRIBUTED AS ERLANG WITH SHAPE PARAMETER 2.
''
''INPUT:
''LAMBDA                ARRIVAL RATE PER INDIVIDUAL IN THE POPULATION
''MU1                   SERVICE RATE OR RECIPROCAL OF MEAN SERVICE TIME
''MPOP                  CUSTOMER POPULATION SIZE
''MLIM                  MAX ALLOWED CUSTOMERS IN THE SERVICE SYSTEM
''NSERVE                NUMBER OF SERVERS (N LE 3)
''MODE                  INTEGER FLAG FOR PRINTING FROM ROUTINE (FOR MODE=1)
''
''OUTPUT:
''P.NSTATE              VECTOR OF PROBABILITY ELEMENTS IN WHICH THE N TH
''                      ELEMENT IS THE PROBABILITY THAT N INDIVIDUALS ARE
''                      IN THE SERVICE SYSTEM IN THE STEADY STATE.
''P.NULL                PROBABILITY THAT THE SYSTEM IS EMPTY
''P.SYS.FULL            PROBABILITY THAT ALL SERVICE CHANNELS ARE FULL
''P.CUST.WAIT           PROBABILITY THAT AN ARRIVING CUSTOMER MUST QUEUE
''ENSYS                 EXPECTED NUMBER OF CUSTOMERS IN THE SYSTEM
''SDNSYS                STD DEVIATION OF THE NUMBER IN THE SYSTEM
''ENQ                   EXPECTED NUMBER OF CUSTOMERS IN THE QUEUE
''ESYS.WAIT             EXPECTED WAIT IN THE SERVICE SYSTEM
''EQ.WAIT               EXPECTED WAIT IN THE SERVICE QUEUE
''EQ.WAIT.GQ            EXPECTED QUEUE WAITING TIME, GIVEN THAT A
''                      CUSTOMER MUST QUEUE.
''SDQ.WAIT.GQ           STANDARD DEVIATION OF QUEUE WAITING TIME, GIVEN
''                      THAT A CUSTOMER MUST QUEUE.
''E.BUSY.SERVERS        EXPECTED VALUE OF BUSY SERVERS
''SD.BUSY.SERVERS       STANDARD DEVIATION OF THE NUMBER OF BUSY SERVERS
''OPTIONAL PRINTING OF THE QUEUE WAITING TIME DISTRIBUTION IS PROVIDED.
''
''ENDOGENOUS VARIABLES:
''AM                    KOLMOGOROV STATE-TRANSITION MATRIX
''BV                    VECTOR OF PROBABILITIES IN WHICH THE N TH
''                      ELEMENT IS THE PROBABILITY OF FINDING THE
''                      SYSTEM IN THE N TH STATE, WHERE
''                      N=2*(NO CUSTOMERS-1)+STAGE OF SERVICE, NO GT 0.
''QV                    VECTOR OF PROBABILITIES IN WHICH THE N TH
''                      ELEMENT IS THE PROBABILITY THAT AN ARRIVING CUSTOMER
''                      FINDS THE SYSTEM IN THE N TH STATE. THIS VECTOR
''                      IS USED TO CALCULATE THE DISTRIBUTION OF
''                      WAIT IN QUEUE.
''IPVT(*)               INTEGER VECTOR USED IN FACTORING THE STATE-TRANSITION
''                      MATRIX.
''DET(*)                TWO-ELEMENT VECTOR USED TO STORE THE DETERMINANT OF
''                      THE STATE-TRANSITION MATRIX.
''
DEFINE I,INFO,J,MLIM,MPOP,TWOM,MAX,MAXM,MODE,N,NSERVE AS INTEGER VARIABLES
```

```
DEFINE IPVT AS AN INTEGER, 1-DIMENSIONAL ARRAY
DEFINE BV,DET,PV,QV,P.NSTATE,AND Q.NSTATE AS REAL,1-DIMENSIONAL ARRAYS
DEFINE AM AS A REAL, 2-DIMENSIONAL ARRAY
    IF MLIM LE 0 OR MLIM  GT  MPOP
        PRINT 1 LINE WITH MPOP,MLIM
        THUS
ERROR IN INPUT TO FINITE.ME2.Q.  MPOP = ***  MLIM = ***.
        STOP
    OTHERWISE
    LET TWOM=2*MLIM
    RESERVE DET(*) AS 2
    LET MU=2.0*MU1 ''SERVICE RATE FOR EACH OF THE 2 STAGES
    LET ML=REAL.F(MPOP)*LAMBDA ''MAX ARRIVAL RATE
    RESERVE P.NSTATE(*) AS MPOP
    RESERVE Q.NSTATE(*) AS MPOP
    IF NSERVE LE 0
        RETURN
    OTHERWISE
    IF NSERVE GT 1
        GO TO LO
    OTHERWISE ''NSERVE=1
    RESERVE PV(*) AS TWOM
    RESERVE QV(*) AS TWOM
    RESERVE AM(*,*) AS TWOM BY TWOM
    RESERVE IPVT(*) AS TWOM
FOR I=1 TO TWOM DO
    LET QV(I)=0.0
    FOR J=1 TO TWOM DO
        LET AM(I,J)=0.0
    LOOP ''OVER J
LOOP ''OVER I TO INITIALIZE
''
''FILL NON-ZERO ELEMENTS OF THE REDUCED STATE-TRANSITION MATRIX (AM)
''AND THE CONSTANT VECTOR (BV).
''
FOR J=1 TO TWOM DO
    LET AM(1,J)=-ML
LOOP ''OVER COLUMNS
    SUBTRACT MU+REAL.F(MPOP-1)*LAMBDA FROM AM(1,1)
    ADD MU TO AM(1,4)
    LET AM(2,1)=MU
    LET AM(2,2)=-MU-ML+LAMBDA
FOR N=2 TO MLIM-1 DO
    LET I=2*N-1
    LET AM(I,I-2)=(MPOP-N+1)*LAMBDA
    LET AM(I,I)=-MU-(MPOP-N)*LAMBDA
    LET AM(I,I+3)=MU
    LET AM(I+1,I-1)=AM(I,I-2)
    LET AM(I+1,I)=MU
    LET AM(I+1,I+1)=AM(I,I)
LOOP ''OVER N
''FINALLY,
    LET AM(TWOM-1,TWOM-3)=LAMBDA*(MPOP-MLIM+1)
    LET AM(TWOM-1,TWOM-1)=-MU
    LET AM(TWOM,TWOM-2)=LAMBDA*(MPOP-MLIM+1)
    LET AM(TWOM,TWOM-1)=MU
    LET AM(TWOM,TWOM)=-MU
```

```
''
''OBTAIN THE INVERSE OF AM(*,*) IN PLACE.
''
''
''FACTOR AM(*,*) FIRST TO OBTAIN AN INVERSE.
''
     CALL SGEFA (AM(*,*),IPVT(*),INFO)
     IF INFO NE 0
          PRINT 1 LINE WITH INFO THUS
TROUBLE FACTORING AM.    INFO = *.
          STOP
     OTHERWISE
     CALL SGEDI (AM(*,*),IPVT(*),DET(*),11)
''
''OBTAIN THE SOLUTION VECTOR OF THE MATRIX EQUATION.
''
     FOR I=1 TO TWOM, LET PV(I)=-ML*AM(I,1)
''
''SOLVE FOR THE EMPTY-SYSTEM PROBABILITY (P.NULL) AND CALCULATE
''THE STATE-PROBABILITY VECTOR (P.NSTATE).
''
     LET ENSYS=0.0
     LET VNSYS=0.0
     LET ENQ=0.0
     LET ESQ=0.0
     LET SUM=0.0
 FOR N=1 TO MLIM DO
     LET I=2*N-1
     LET P.NSTATE(N)=PV(I)+PV(I+1)
     ADD P.NSTATE(N) TO SUM
     ADD N*P.NSTATE(N) TO ENSYS
     ADD N**2*P.NSTATE(N) TO VNSYS
     ADD (N-1)*P.NSTATE(N) TO ENQ
     ADD (N-1)**2*P.NSTATE(N) TO ESQ
 LOOP ''OVER NON-ZERO SYSTEM STATES
''
''CHECK VALIDITY OF PROBABILITY SUM.
''
     IF SUM LT 0.0 OR SUM GT 1.0
          PRINT 1 LINE WITH SUM THUS
ERROR IN ROUTINE FINITE.ME2.Q.  PARTIAL SUM OF STATE PROBS = **.***.
          STOP
     OTHERWISE
     LET P.NULL=1.0-SUM
     LET P.SYS.FULL=SUM ''FOR A SINGLE SERVER
''
''CALCULATE THE VARIANCE AND STD DEV OF THE NUMBER IN THE SYSTEM.
''
     LET VNSYS=VNSYS-ENSYS**2
     LET SDNSYS=SQRT.F(VNSYS)
     LET VNQ=ESQ-ENQ**2
     LET SDNQ=SQRT.F(VNQ)
''
''CALCULATE THE MEAN AND STANDARD DEVIATION OF BUSY SERVERS.
''
     LET E.BUSY.SERVERS=1.0-P.NULL ''FOR A SINGLE SERVER
     LET VAR.BUSY.SERVERS=1.0-P.NULL-E.BUSY.SERVERS**2
     LET SD.BUSY.SERVERS=SQRT.F(VAR.BUSY.SERVERS)
```

```
''
''CALCULATE MEAN WAITING TIMES USING LITTLE'S FORMULA.
''
        LET ESYS.WAIT=ENSYS/LAMBDA/(REAL.F(MPOP)-ENSYS)
        LET EQ.WAIT=ESYS.WAIT-1.0/MU1
        IF MPOP=1
            RETURN
        OTHERWISE ''CALCULATE AND PRINT WAITING-TIME DISTRIBUTIONS
''
''CALCULATE THE PROBABILITY THAT AN ARRIVAL FINDS THE SYSTEM
''IN THE N TH STATE (QV(N)).
''
        LET NORM.CONST=MPOP*P.NULL
    FOR N=1 TO MLIM-1 DO
        ADD (MPOP-N)*P.NSTATE(N) TO NORM.CONST
    LOOP ''TO CALCULATE THE NORMALIZATION CONSTANT
        LET Q.NULL=MPOP*P.NULL/NORM.CONST
        LET P.CUST.WAIT=1.0-Q.NULL
        LET EQ.WAIT.GQ=0.0
        LET VQ.WAIT.GQ=0.0
    FOR N=1 TO MLIM-1 DO
        LET I=2*N-1
        LET QV(I)=(MPOP-N)*PV(I)/NORM.CONST
        LET QV(I+1)=(MPOP-N)*PV(I+1)/NORM.CONST
        LET Q.NSTATE(N)=QV(I)+QV(I+1)
        ADD (I+1)*QV(I)+I*QV(I+1) TO EQ.WAIT.GQ
        ADD (I+1)*(I+2)*QV(I)+I*(I+1)*QV(I+1) TO VQ.WAIT.GQ
    LOOP ''OVER THE NUMBER OF ARRIVING CUSTOMERS
            LET EQ.WAIT.CHECK=EQ.WAIT.GQ./MU
            LET ESYS.WAIT.CHECK=EQ.WAIT.CHECK +1.0/MU1
            LET EQ.WAIT.GQ=EQ.WAIT.GQ/MU/(1.0-Q.NULL)
            LET VQ.WAIT.GQ=VQ.WAIT.GQ/MU/MU/(1.0-Q.NULL)-EQ.WAIT.GQ**2
            LET SDQ.WAIT.GQ=SQRT.F(VQ.WAIT.GQ)
            IF MODE NE 1
                RETURN
            OTHERWISE
    'L1'SKIP 4 LINES
        PRINT 21 LINES WITH NSERVE,P.SYS.FULL,ENSYS,SDNSYS,ENQ,SDNQ,E.BUSY.SERVERS,
        SD.BUSY.SERVERS,ESYS.WAIT,EQ.WAIT,EQ.WAIT.GQ,P.NULL,P.NULL,Q.NULL,Q.NULL
        THUS
  STATE OF THE SYSTEM IN STEADY STATE WITH GAMMA(2) SERVICE FOR ** SERVERS

  PROBABILITY:  SYSTEM  IS FULL      *.****
  EXPECTED NUMBER IN THE SYSTEM     **.****
  STD DEV  NUMBER IN THE SYSTEM     **.****
  EXPECTED NUMBER IN THE  QUEUE     **.****
  STD DEV  NUMBER IN THE  QUEUE     **.****
  EXPECTED NUMBER  BUSY SERVERS     **.****
  STD DEV  NUMBER  BUSY SERVERS     **.****
  MEAN WAITING TIME (HRS) IN SYSTEM    ***.***
  MEAN WAITING TIME (HRS) IN QUEUE     ***.***
  COND'AL MEAN TIME (HRS) IN QUEUE     ***.***

  PROBABILITY DISTRIBUTIONS FOR NUMBER OF CUSTOMERS IN THE SYSTEM

  ---------------------------------------------------------------------------
              UNCONDITIONAL               GIVEN ARRIVAL
  NO IN       PROB       CUML             PROB       CUML
  SYSTEM      DENS       PROB             DENS       PROB

  ---------------------------------------------------------------------------
```

```
 0          *.****    *.****        *.****    *.****
     LET CUM.DIST=P.NULL
     LET CUM.DIST.COND=Q.NULL
 FOR N=1 TO MLIM DO
     ADD P.NSTATE(N) TO CUM.DIST
     ADD Q.NSTATE(N) TO CUM.DIST.COND
     PRINT 1 LINE WITH N, P.NSTATE(N), CUM.DIST, Q.NSTATE(N), CUM.DIST.COND
     THUS
 **         *.****    *.****        *.****    *.****
     IF CUM.DIST GE 0.9999
         GO TO K1
     OTHERWISE
 LOOP ''OVER THE SYSTEM STATES
 'K1'PRINT 2 LINES THUS
---------------------------------------------------------------------------

     IF NSERVE GT 1
         RETURN
     OTHERWISE
     LET DELT=MAX.F(1.0,TRUNC.F(EQ.WAIT.GQ/10.0))
     LET MAX=60
     SKIP 2 LINES
 ''
 ''PRINT HEADINGS FOR THE QUEUE WAITING TIME DISTRIBUTION.
 ''
     PRINT 8 LINES WITH MPOP AND Q.NULL THUS
CUM AND CONDITIONAL CUM PROB DISTRIBUTIONS OF WAITING TIME IN QUEUE
EXPONENTIAL INTERARRIVALS FOR EACH OF ** CUSTOMERS.ERLANG(2) SERVICE.


---------------------------------------------------------------------------
 TIME  CUML    COND
(HRS)  PROB    PROB

---------------------------------------------------------------------------
0     *.****    0
 ''
 ''START TIME LOOP.
 ''
     FOR I=1 TO MAX ''TIME STEPS'' DO
         LET TM=I*DELT
         LET ARG=TM*MU
         LET CUM.DIST=1.0-EXP.F(-ARG)*(QV(1)*(1.0+ARG)+QV(2))
         LET SUMN=0.0
         FOR N=2 TO MLIM-1 DO
             LET SUMEJ=1.0
             LET ARGJ=1.0
             LET JFACTORIAL=1.0
             FOR J=1 TO 2*(N-1) DO
                 LET ARGJ=ARGJ*ARG
                 LET JFACTORIAL=JFACTORIAL*J
                 ADD ARGJ/JFACTORIAL TO SUMEJ
             LOOP ''OVER J
             LET ARGJ=ARGJ*ARG
             LET JFACTORIAL=JFACTORIAL*(2*N-1)
             LET SUMOJ=SUMEJ+ARGJ/JFACTORIAL
             ADD QV(2*N-1)*SUMOJ+QV(2*N)*SUMEJ TO SUMN
         LOOP ''OVER N
         LET CUM.DIST=CUM.DIST-EXP.F(-ARG)*SUMN
```

```
          LET CON.DIST=(CUM.DIST-Q.NULL)/(1.0-Q.NULL)
          PRINT 1 LINE WITH TM, CUM.DIST, AND CON.DIST THUS
***.* *.****  *.****
          IF CUM.DIST GE 0.9999
              GO TO K2
          OTHERWISE
      LOOP ''OVER I TIME STEPS
 'K2'PRINT 10 LINES WITH EQ.WAIT.GQ, SDQ.WAIT.GQ, EQ.WAIT.CHECK,
      ESYS.WAIT.CHECK, Q.NULL, E.BUSY.SERVERS, SD.BUSY.SERVERS
      THUS
-----------------------------------------------------------------------------------

MEAN  WAITING TIME IN QUEUE, GIVEN A WAIT     ***.***
STD DEV WAITING TIME IN QUEUE, GIVEN A WAIT   ***.***
MEAN  WAITING TIME IN QUEUE (UNCONDITIONAL)   ***.***
MEAN  WAITING TIME IN SYS   (UNCONDITIONAL)   ***.***
PROBABILITY THAT AN ARRIVAL FINDS SYS EMPTY    *.****
MEAN  NUMBER OF BUSY SERVERS _____________    **.****
STD DEV NUMBER OF BUSY SERVERS _____________  **.****

      RETURN
 'LO'IF NSERVE GT 2
          GO TO L3
      OTHERWISE
 ''
 ''RESERVE ARRAYS FOR THE CASE:  NSERVE=2
 ''
      LET MAXM=3*MLIM-1
 ''   RESERVE BV(*) AS MAXM
      RESERVE PV(*) AS MAXM
      RESERVE QV(*) AS MAXM
      RESERVE AM(*,*) AS MAXM BY MAXM
      RESERVE IPVT(*) AS MAXM
 ''
 ''FILL THE REDUCED STATE-TRANSITION MATRIX (AM).
 ''
 FOR I=1 TO MAXM DO
     FOR J=1 TO MAXM DO
         LET AM(I,J)=0.0
     LOOP ''OVER J
 LOOP ''OVER I
 FOR J=1 TO MAXM DO
     LET AM(1,J)=ML
 LOOP ''OVER J
     LET AM(1,2)=AM(1,2)+MU
     LET AM(2,1)=MU
     LET AM(2,2)=-(MPOP-1)*LAMBDA-MU
     LET AM(2,5)=2.0*MU
     LET AM(3,1)=(MPOP-1)*LAMBDA
     LET AM(3,3)=-(MPOP-2)*LAMBDA-2.0*MU
     LET AM(3,7)=MU
     LET AM(4,2)=(MPOP-1)*LAMBDA
     LET AM(4,3)=2.0*MU
     LET AM(4,4)=AM(3,3)
     LET AM(4,8)=2.0*MU
     LET AM(5,4)=MU
     LET AM(5,5)=AM(4,4)
```

```
FOR N=3 TO MLIM-1 DO
    LET I=3*(N-1)
    LET AM(I,3*N-6)=(MPOP-N+1)*LAMBDA
    LET AM(I,I)=-((MPOP-N)*LAMBDA+2.0*MU)
    LET AM(I,3*N+1)=MU
    ADD 1 TO I
    LET AM(I,3*N-5)=(MPOP-N+1)*LAMBDA
    LET AM(I,3*N-3)=2.0*MU
    LET AM(I,I)=AM(I-1,I-1)
    LET AM(I,3*N+2)=2.0*MU
    ADD 1 TO I
    LET AM(I,3*N-4)=(MPOP-N+1)*LAMBDA
    LET AM(I,3*N-2)=MU
    LET AM(I,I)=-((MPOP-N)*LAMBDA+2.0*MU)
LOOP ''OVER NUMBER OF CUSTOMERS IN THE SYSTEM
    LET I=3*(MLIM-1)
    LET AM(I,3*MLIM-6)=LAMBDA*(MPOP-MLIM+1)
    LET AM(I,I)=-2.0*MU
    ADD 1 TO I
    LET AM(I,3*MLIM-5)=LAMBDA*(MPOP-MLIM+1)
    LET AM(I,3*MLIM-3)=2.0*MU
    LET AM(I,I)=-2.0*MU
    ADD 1 TO I
    LET AM(I,3*MLIM-4)=LAMBDA*(MPOP-MLIM+1)
    LET AM(I,3*MLIM-2)=MU
    LET AM(I,I)=-2.0*MU
''
''OBTAIN THE INVERSE OF AM(*,*) IN PLACE.
''
''
''FACTOR AM(*,*).
''
    CALL SGEFA (AM(*,*),IPVT(*),INFO)
    IF INFO NE 0
        PRINT 1 LINE WITH INFO THUS
TROUBLE FACTORING AM.  INFO = *.
        STOP
    OTHERWISE
    CALL SGEDI (AM(*,*),IPVT(*),DET(*),11)
''
''OBTAIN THE SOLUTION VECTOR OF THE MATRIX EQUATION.
''
    FOR I=1 TO MAXM, LET PV(I)=ML*AM(I,1)
''
''SOLVE FOR THE EMPTY-SYSTEM PROBABILITY (P.NULL) AND CALCULATE THE
''CUSTOMER STATE PROBABILITY VECTOR (P.NSTATE).
''
    LET ENSYS=PV(1)+PV(2)
    LET P.NSTATE(1)=ENSYS
    LET VNSYS=ENSYS
    LET ENQ=0.0
    LET ESQ=0.0
    LET SUM=ENSYS
FOR N=2 TO MLIM DO
    LET I=3*(N-1)
```

```
      LET P.NSTATE(N)=PV(I)+PV(I+1)+PV(I+2)
      ADD P.NSTATE(N) TO SUM
      ADD N*P.NSTATE(N) TO ENSYS
      ADD (N-2)*P.NSTATE(N) TO ENQ
      ADD (N-2)**2*P.NSTATE(N) TO ESQ
      ADD N**2*P.NSTATE(N) TO VNSYS
  LOOP ''OVER THE OTHER NON-ZERO SYSTEM STATES
  ''
  ''CHECK VALIDITY OF THE PROBABILITY SUM.
  ''
      IF SUM LT 0.0 OR SUM GT 1.0
          PRINT 1 LINE WITH SUM THUS
ERROR IN ROUTINE FINITE.ME2.Q.  PARTIAL SUM OF STATE PROBS = **.****.
          STOP
      OTHERWISE
      LET P.NULL=1.0-SUM
      LET P.SYS.FULL=1.0-P.NULL-P.NSTATE(1) ''FOR 2 SERVERS
  ''
  ''CALCULATE THE VARIANCE AND STANDARD DEV OF THE NUMBER IN THE SYSTEM.
  ''
      LET VNSYS=VNSYS-ENSYS**2
      LET SDNSYS=SQRT.F(VNSYS)
      LET VNQ=ESQ-ENQ**2
      LET SDNQ=SQRT.F(VNQ)
  ''
  ''CALCULATE THE MEAN AND STANDARD DEVIATION OF BUSY SERVERS.
  ''
      LET E.BUSY.SERVERS=P.NSTATE(1)+2.0*(1.0-P.NULL-P.NSTATE(1))
      LET VAR.BUSY.SERVERS=P.NSTATE(1)+4.0*(1.0-P.NULL-P.NSTATE(1))
          -E.BUSY.SERVERS**2
      LET SD.BUSY.SERVERS=SQRT.F(VAR.BUSY.SERVERS)
  ''
  ''CALCULATE MEAN WAITING TIME USING LITTLE'S FORMULA.
  ''
      LET ESYS.WAIT=ENSYS/LAMBDA/(REAL.F(MPOP)-ENSYS)
      LET EQ.WAIT=ENQ/LAMBDA/(REAL.F(MPOP)-ENSYS)
      IF MPOP LE 2
          RETURN
      OTHERWISE ''CALCULATE AND PRINT CONDITIONAL PROB DISTRIBUTIONS
  ''
  ''CALCULATE THE PROBABILITY THAT AN ARRIVAL FINDS THE SYSTEM IN STATE N.
  ''
      LET NORM.CONST=MPOP*P.NULL
  FOR N=1 TO MLIM-1 DO
      ADD (MPOP-N)*P.NSTATE(N) TO NORM.CONST
  LOOP ''TO CALCULATE THE NORMALIZATION CONSTANT
      LET Q.NULL=MPOP*P.NULL/NORM.CONST
      LET QV(1)=(MPOP-1)*PV(1)/NORM.CONST
      LET QV(2)=(MPOP-1)*PV(2)/NORM.CONST
      LET Q.NSTATE(1)=QV(1)+QV(2)
  FOR N=2 TO MLIM-1 DO
      LET I=3*(N-1)
      LET QV(I)=(MPOP-N)*PV(I)/NORM.CONST
      LET QV(I+1)=(MPOP-N)*PV(I+1)/NORM.CONST
      LET QV(I+2)=(MPOP-N)*PV(I+2)/NORM.CONST
      LET Q.NSTATE(N)=QV(I)+QV(I+1)+QV(I+2)
  LOOP ''OVER NUMBER OF ARRIVING CUSTOMERS
```

```
      LET P.CUST.WAIT=1.0-Q.NULL-Q.NSTATE(1)
 'L2'LET EQ.WAIT.GQ=EQ.WAIT/P.CUST.WAIT
      LET SDQ.WAIT.GQ=EQ.WAIT.GQ ''APPROXIMATELY
      IF MODE NE 1 RETURN
      OTHERWISE
      GO TO L1
 'L3'IF MPOP LE 3
          PRINT 1 LINE WITH MPOP AND NSERVE THUS
ERROR IN FINITE.ME2.Q.  MPOP = **.  NSERVE = **.
          STOP
      OTHERWISE
 ''
 ''RESERVE ARRAYS FOR THE CASE:  NSERVE = 3.
 ''
      LET MAXM=4*MLIM-3
      RESERVE PV(*) AS MAXM
      RESERVE QV(*) AS MAXM
      RESERVE AM(*,*) AS MAXM BY MAXM
      RESERVE IPVT(*) AS MAXM
 ''
 ''FILL THE REDUCED STATE TRANSITION MATRIX (AM) .
 ''
 FOR I=1 TO MAXM DO
      FOR J=1 TO MAXM DO
          LET AM(I,J)=0.0
      LOOP ''OVER J
 LOOP ''OVER I
 FOR J=1 TO MAXM DO
      LET AM(1,J)=ML
 LOOP ''OVER J
      LET AM(1,2)=AM(1,2)+MU
      LET AM(2,1)=MU
      LET AM(2,2)=-((MPOP-1)*LAMBDA+MU)
      LET AM(2,5)=2.0*MU
      LET AM(3,1)=(MPOP-1)*LAMBDA
      LET AM(3,3)=-((MPOP-2)*LAMBDA+2.0*MU)
      LET AM(3,7)=MU
      LET AM(4,2)=(MPOP-1)*LAMBDA
      LET AM(4,3)=2.0*MU
      LET AM(4,4)=AM(3,3)
      LET AM(4,8)=2.0*MU
      LET AM(5,4)=MU
      LET AM(5,5)=AM(4,4)
      LET AM(5,9)=3.0*MU
      LET AM(6,3)=(MPOP-2)*LAMBDA
      LET AM(6,6)=-((MPOP-3)*LAMBDA+3.0*MU)
      LET AM(6,11)=MU
      LET AM(7,4)=AM(6,3)
      LET AM(7,6)=3.0*MU
      LET AM(7,7)=AM(6,6)
      LET AM(7,12)=2.0*MU
      LET AM(8,5)=AM(7,4)
      LET AM(8,7)=2.0*MU
      LET AM(8,8)=AM(7,7)
      LET AM(8,13)=3.0*MU
      LET AM(9,8)=MU
      LET AM(9,9)=AM(8,8)
```

```
FOR N=4 TO MLIM-1 DO
     LET I=4*N-6 ''AS THE ROW INDEX
     LET AM(I,4*N-10)=(MPOP-N+1)*LAMBDA
     LET AM(I,I)=-((MPOP-N)*LAMBDA+3.0*MU)
     LET AM(I,4*N-1)=MU
     ADD 1 TO I ''I=4*N-5
     LET AM(I,4*N-9)=(MPOP-N+1)*LAMBDA
     LET AM(I,I-1)=3.0*MU
     LET AM(I,I)=AM(I-1,I-1)
     LET AM(I,4*N)=2.0*MU
     ADD 1 TO I ''I=4*N-4
     LET AM(I,4*N-8)=(MPOP-N+1)*LAMBDA
     LET AM(I,I-1)=2.0*MU
     LET AM(I,I)=AM(I-1,I-1)
     LET AM(I,4*N+1)=3.0*MU
     ADD 1 TO I ''I=4*N-3
     LET AM(I,4*N-7)=(MPOP-N+1)*LAMBDA
     LET AM(I,I-1)=MU
     LET AM(I,I)=AM(I-1,I-1)
LOOP ''OVER THE NUMBER OF CUSTOMERS IN THE SYSTEM
''
''EQUATIONS FOR THE LAST FOUR STATES.
''
     LET I=4*MLIM-6
     LET AM(I,4*MLIM-10)=LAMBDA*(MPOP-MLIM+1)
     LET AM(I,I)=-3.0*MU
     ADD 1 TO I ''I=4*MLIM-5
     LET AM(I,4*MLIM-9)=LAMBDA*(MPOP-MLIM+1)
     LET AM(I,I-1)=3.0*MU
     LET AM(I,I)=AM(I-1,I-1)
     ADD 1 TO I ''I=4*MLIM-4
     LET AM(I,4*MLIM-8)=LAMBDA*(MPOP-MLIM+1)
     LET AM(I,I-1)=2.0*MU
     LET AM(I,I)=AM(I-1,I-1)
     ADD 1 TO I ''I=4*MLIM-3
     LET AM(I,4*MLIM-7)=LAMBDA*(MPOP-MLIM+1)
     LET AM(I,I-1)=MU
     LET AM(I,I)=AM(I-1,I-1)
''
''OBTAIN THE INVERSE OF AM(*,*) IN PLACE.
''
''FACTOR AM(*,*).
''
     CALL SGEFA (AM(*,*),IPVT(*),INFO)
     IF INFO NE 0
          PRINT 1 LINE WITH INFO THUS
TROUBLE FACTORING AM.  INFO = *.
          STOP
     OTHERWISE
     CALL SGEDI (AM(*,*),IPVT(*),DET(*),11)
''
''OBTAIN THE SOLUTION VECTOR OF THE MATRIX EQUATION.
''
     FOR I=1 TO MAXM, LET PV(I)=ML*AM(I,1)
```

```
''
''SOLVE FOR THE EMPTY SYSTEM STATE PROBABILITY (P.NULL) AND CALCULATE
''THE CUSTOMER STATE PROBABILITY VECTOR (P.NSTATE(*)).
''
     LET P.NSTATE(1)=PV(1)+PV(2) ''FOR 1 CUSTOMER
     LET P.NSTATE(2)=PV(3)+PV(4)+PV(5)
     LET ENSYS=P.NSTATE(1)+2.0*P.NSTATE(2)
     LET VNSYS=P.NSTATE(1)+4.0*P.NSTATE(2)
     LET ENQ=0.0
     LET ESQ=0.0
     LET SUM=P.NSTATE(1)+P.NSTATE(2)
FOR N=3 TO MLIM DO
     LET I=4*N-6
     LET P.NSTATE(N)=PV(I)+PV(I+1)+PV(I+2)+PV(I+3)
     ADD P.NSTATE(N) TO SUM
     ADD N*P.NSTATE(N) TO ENSYS
     ADD (N-3)*P.NSTATE(N) TO ENQ
     ADD (N-3)**2*P.NSTATE(N) TO ESQ
     ADD N**2*P.NSTATE(N) TO VNSYS
LOOP ''OVER THE OTHER NON-ZERO STATES
''
''CHECK VALIDITY OF THE PROBABILITY SUM.
''
     IF SUM LT 0.0 OR SUM GT 1.0
          PRINT 1 LINE WITH SUM THUS
ERROR IN ROUTINE FINITE.ME2.Q.  PARTIAL SUM OF STATE PROBS = **.****.
          STOP
     OTHERWISE
     LET P.NULL=1.0-SUM
     LET P.SYS.FULL=1.0-P.NULL-P.NSTATE(1)-P.NSTATE(2) ''FOR 3 SERVERS
''
''CALCULATE THE VARIANCE AND STANDARD DEVIATION OF NUMBER IN THE SYSTEM.
''
     LET VNSYS=VNSYS-ENSYS**2
     LET SDNSYS=SQRT.F(VNSYS)
     LET VNQ=ESQ-ENQ**2
     LET SDNQ=SQRT.F(VNQ)
''
''CALCULATE THE MEAN AND STANDARD DEVIATION OF BUSY SERVERS.
''
     LET E.BUSY.SERVERS=P.NSTATE(1)+2.0*P.NSTATE(2)+3.0*(1.0-P.NULL
          -P.NSTATE(1)-P.NSTATE(2))
     LET VAR.BUSY.SERVERS=P.NSTATE(1)+4.0*P.NSTATE(2)+9.0*(1.0-P.NULL
          -P.NSTATE(1)-P.NSTATE(2))-E.BUSY.SERVERS**2
     LET SD.BUSY.SERVERS=SQRT.F(VAR.BUSY.SERVERS)
''
''CALCULATE MEAN WAITING TIME USING LITTLE'S FORMULA.
''
     LET ESYS.WAIT=ENSYS/LAMBDA/(REAL.F(MPOP)-ENSYS)
     LET EQ.WAIT=ENQ/LAMBDA/(REAL.F(MPOP)-ENSYS)
     IF MPOP=3
          RETURN
     OTHERWISE ''CALCULATE AND PRINT CONDITIONAL PROB DISTRIBUTIONS
```

```
''
''CALCULATE THE PROBABILITY THAT AN ARRIVAL FINDS THE SYSTEM IN STATE N.
''
     LET NORM.CONST=MPOP*P.NULL
FOR N=1 TO MLIM-1 DO
     ADD (MPOP-N)*P.NSTATE(N) TO NORM.CONST
LOOP ''TO CALCULATE THE NORMALIZATION CONSTANT
     LET Q.NULL=MPOP*P.NULL/NORM.CONST
     LET QV(1)=(MPOP-1)*PV(1)/NORM.CONST
     LET QV(2)=(MPOP-1)*PV(2)/NORM.CONST
     LET QV(3)=(MPOP-2)*PV(3)/NORM.CONST
     LET QV(4)=(MPOP-2)*PV(4)/NORM.CONST
     LET QV(5)=(MPOP-2)*PV(5)/NORM.CONST
     LET Q.NSTATE(1)=QV(1)+QV(2)
     LET Q.NSTATE(2)=QV(3)+QV(4)+QV(5)
FOR N=3 TO MLIM-1 DO
     LET I=4*N-6
     LET QV(I)=(MPOP-N)*PV(I)/NORM.CONST
     LET QV(I+1)=(MPOP-N)*PV(I+1)/NORM.CONST
     LET QV(I+2)=(MPOP-N)*PV(I+2)/NORM.CONST
     LET QV(I+3)=(MPOP-N)*PV(I+3)/NORM.CONST
     LET Q.NSTATE(N)=QV(I)+QV(I+1)+QV(I+2)+QV(I+3)
LOOP ''OVER NUMBER OF ARRIVING CUSTOMERS
     LET P.CUST.WAIT=1.0-Q.NULL-Q.NSTATE(1)-Q.NSTATE(2)
     GO TO L2
END ''ROUTINE FINITE.ME2.Q
```

```
ROUTINE SGEFA ( A, IPVT, INFO)
''
''ROUTINE FACTORS THE MATRIX A(*,*) INTO UPPER (U) AND STRICTLY LOWER (L)
''TRIANGULAR MATRICES SUCH THAT A(*,*) = U(*,*)L(*,*).
''ROUTINE IS INTENDED FOR USE WITH OTHER ROUTINES OF THE LINEAR OPERATIONS
''PACKAGE--LINPACK.  THIS VERSION IS A CONVERSION OF THE FORTRAN ROUTINE
''WRITTEN BY CLEVE MOLER, U. OF N.M. AND ARGONNE NAT LAB.
''
''ARGUMENTS:
''NAME        MODE        ENTRY VALUE                RETURN VALUE
''
''A           REAL(N, N) SQUARE MATRIX.       AN UPPER TRIANGULAR MATRIX AND
''                                            THE MULTIPLIERS WHICH WERE USED TO
''                                            GET IT.  THESE ARE STORED IN L.
''N           INTEGER ORDER OF THE MATRIX A.  DIMENSION OF A(*,*).
''IPVT        INTEGER(N).                     VECTOR OF PIVOT INDICES.
''INFO        INTEGER INDICATOR.              = 0 FOR NORMAL VALUE.
''                                            = K IF U(K,K) EQ 0.0.  THIS
''                                            INDICATES THAT SGESL OR SGEDI
''                                            WILL DIVIDE BY ZERO IF CALLED.
''
    DEFINE I,INFO,J,K,KP1,L,N,NM1 AS INTEGER VARIABLES
    DEFINE IPVT AS AN INTEGER, 1-DIMENSIONAL ARRAY
    DEFINE A AS A REAL, 2-DIMENSIONAL ARRAY
''
''GAUSSIAN ELIMINATION WITH PARTIAL PIVOTING.
''
    LET N=DIM.F(IPVT(*))
    LET INFO=0
    LET NM1=N-1
    IF NM1  LT  1
        GO TO L7
    OTHERWISE
    FOR K=1 TO NM1 DO
        LET KP1=K+1
''
''FIND L = PIVOT INDEX IN THIS COLUMN.
''
        LET SMAX=ABS.F(A(K,K))
        LET L=K
        FOR I=K+1 TO N DO
            IF ABS.F(A(I,K))  GT  SMAX
                LET L=I
                LET SMAX=ABS.F(A(I,K))
            ALWAYS
        LOOP ''FOR MAX ELEMENT
        LET IPVT(K)=L
''
''ZERO PIVOT IMPLIES THIS COLUMN ALREADY TRIANGULARIZED.
''
        IF A(L,K) = 0.0
            GO TO L4
        OTHERWISE
''
''INTERCHANGE IF NECESSARY.
''
```

```
          IF L = K
              GO TO L1
          OTHERWISE
          LET T=A(L,K)
          LET A(L,K)=A(K,K)
          LET A(K,K)=T
'L1'      LET T=-1.0/A(K,K)
          FOR I=K+1 TO N, LET A(I,K)=T*A(I,K)
''
''ROW ELIMINATION WITH COLUMN INDEXING.
''
          FOR J=KP1 TO N DO
              LET T=A(L,J)
              IF L=K
                  GO TO L2
              OTHERWISE
              LET A(L,J)=A(K,J)
              LET A(K,J)=T
'L2'          FOR I=K+1 TO N, LET A(I,J)=T*A(I,K)+A(I,J)
          LOOP ''OVER (J) COLUMNS
          GO TO L5
'L4'      LET INFO=K
'L5'LOOP ''OVER K
'L7'LET IPVT(N)=N
     IF A(N,N)=0.0
         LET INFO=N
     ALWAYS
     RETURN
END ''SGEFA
```

```
ROUTINE SGEDI (A, IPVT, DET, JOB)
''
''
''SGEDI COMPUTES THE DETERMINANT AND INVERSE OF A MATRIX USING THE RESULTS
''PRODUCED BY SGEFA.
''
''ARGUMENTS:
''A(*,*) _ THE REAL FACTORED MATRIX FROM SGEFA ON INPUT.  ON OUTPUT THE
''          ARRAY CONTAINS THE MATRIX INV, IF REQUESTED. OTHERWISE UNCHANGED.
''IPVT(*)  THE INTEGER PIVOT VECTOR FROM SGEFA.
''JOB ____ AN INTEGER SWITCH.
''          = 11 FOR BOTH DETERMINANT AND INVERSE.
''          = 01 FOR INVERSE ONLY.
''          = 10 FOR DETERMINANT ONLY.
''DET(*) _ CONTAINS THE DETERMINANT OF THE MATRIX, IF REQUESTED.  OTHERWISE
''          IS NOT REFERENCED.  THE DETERMINANT = DET(1)*10.0**DET(2), WITH
''          DET(1) BETWEEN 0 AND 10, AND WITH DET(2) A FLOATED INTEGER.
''
''NOTE:  A DIVISION BY ZERO WILL OCCUR IF THE INPUT FACTOR CONTAINS A
''ZERO ON THE DIAGONAL AND THE INVERSE IS REQUESTED.
''
    DEFINE I,J,JOB,K,KB,KP1,L,N,NM1 AS INTEGER VARIABLES
    DEFINE IPVT AS AN INTEGER, 1-DIMENSIONAL ARRAY
    DEFINE DET, WORK AS REAL, 1-DIMENSIONAL ARRAYS
    DEFINE A AS A REAL, 2-DIMENSIONAL ARRAY
    LET N=DIM.F(IPVT(*))
    RESERVE WORK(*) AS N ''LOCALLY
''
''CALCULATE THE DETERMINANT IF REQUESTED.
''
    IF DIV.F(JOB,10) = 0
        GO TO L6
    OTHERWISE
    LET DET(1)=1.0
    LET DET(2)=0.0
    LET TEN=10.0
    FOR I=1 TO N DO
        IF IPVT(I) NE I
            LET DET(1) = -DET(1)
        ALWAYS
        LET DET(1)=A(I,I)*DET(1)
        IF DET(1)=0.0
            GO TO L6
        OTHERWISE
'L1'    IF ABS.F(DET(1)) GE 1.0
            GO TO L2
        OTHERWISE
        LET DET(1)=TEN*DET(1)
        SUBTRACT 1.0 FROM DET(2)
        GO TO L1
'L2'    IF ABS.F(DET(1))  LT  TEN
            GO TO L4
        OTHERWISE
        LET DET(1)=DET(1)/TEN
        ADD 1.0 TO DET(2)
        GO TO L2
'L4'LOOP ''OVER I
```

```
''
''GET INVERSE OF UPPER TRIANGULAR MATRIX U(*,*).
''
'L6'IF MOD.F(JOB,10)=0
          RELEASE WORK(*)
          RETURN
     OTHERWISE
     FOR K=1 TO N DO
          LET A(K,K)=1.0/A(K,K)
          LET T=-A(K,K)
          FOR I=1 TO K-1, LET A(I,K)=T*A(I,K)
          LET KP1=K+1
          IF N  LT  KP1
               GO TO L9
          OTHERWISE
          FOR J=KP1 TO N DO
               LET T=A(K,J)
               LET A(K,J)=0.0
               FOR I=1 TO K, LET A(I,J)=T*A(I,K)+A(I,J)
          LOOP ''OVER J
'L9'LOOP ''OVER K
''
''FORM INVERSE(U)*INVERSE(L)
''
     LET NM1=N-1
     IF NM1  LT  1
          RELEASE WORK(*)
          RETURN
     OTHERWISE
     FOR KB=1 TO NM1 DO
          LET K=N-KB
          LET KP1=K+1
          FOR I=KP1 TO N DO
               LET WORK(I)=A(I,K)
               LET A(I,K)=0.0
          LOOP ''OVER I
          FOR J=KP1 TO N DO
               LET T=WORK(J)
               FOR I=1 TO N, LET A(I,K)=T*A(I,J)+A(I,K)
          LOOP ''OVER J
          LET L=IPVT(K)
          IF L NE K ''SWAP ELEMENTS OF VECTORS K AND L
               FOR I=1 TO N DO
                    LET T=A(I,K)
                    LET A(I,K)=A(I,L)
                    LET A(I,L)=T
               LOOP ''OVER I TO SWAP
          ALWAYS
     LOOP ''OVER KB
     RELEASE WORK(*)
     RETURN
END ''SGEDI
```

```
ROUTINE MINRV (IPRINT,N,XRANGE,APARM,BPARM) YIELDING EMINX,VMINX
''
''ROUTINE CALCULATES THE MEAN AND VARIANCE OF THE MINIMUM OF N CONTINUOUS,
''POSITIVE, I.I.D. RANDOM VARIABLES.  THE C.D.F. FOR THESE R.V.'S IS
''CALLED "FUN.CDF" HERE, AND IS SPECIFIED BY A DEFINE-TO-MEAN STATEMENT
''IN THE PREAMBLE.  IF THE PRINT FLAG IPRINT =1, THE BASIS C.D.F. AND
''C.D.F. OF THE MINIMUM ARE PRINTED FROM THIS ROUTINE.
''
''INPUTS:
''IPRINT ___ INTEGER FLAG TO PRINT FROM THE ROUTINE ( = 1).
''N ________ NUMBER OF I.I.D. RANDOM (X- ) VARIABLES IN THE SET.
''XRANGE ___ UPPER LIMIT ON RANGE OF X, USED TO CALCULATE INTEGRATION STEP.
''APARM ____ FIRST (REAL-VALUED) PARAMETER OF THE C.D.F. OF X.
''BPARM ____ 2 ND  (REAL-VALUED) PARAMETER OF THE C.D.F. OF X.
''
''OUTPUTS:
''EMINX ____ EXPECTED VALUE OF THE MIN OF N RANDOM X-VARIABLES.
''VMINX ____ VARIANCE OF THE MIN X.
''
    DEFINE I,IPRINT,J,K,M,N AS INTEGER VARIABLES
    DEFINE FV,SUMV AS REAL, 1-DIMENSIONAL ARRAYS
    RESERVE FV(*),SUMV(*) AS 2
    LET M=2000 ''STEPS TO INTEGRATE ALLOWED
    LET DELX=XRANGE/M ''INTEGRATION STEP SIZE (SIMPSON'S RULE)
    IF IPRINT NE 1
        GO TO LO
    OTHERWISE
    SKIP 2 LINES
    PRINT 7 LINES WITH N,APARM,BPARM
    THUS
PROB DISTRIBUTION FOR THE SMALLEST OF ** CONTINUOUS RANDOM VARIABLES

PARAMETERS OF BASIS DISTRIBUTION:  1ST =  ............  2ND =  ............
-----------------------------------------------------------------------
  ARGU-     BASIS     CDF OF
  MENT      C.D.F.    MIN SET

-----------------------------------------------------------------------
'LO'LET FV(1)=1.0
    LET FV(2)=0.0
    FOR J=1 TO 2, LET SUMV(J)=FV(J)
    FOR I=1 TO M DO
        LET X=I*DELX
        IF MOD.F(I,2)=0
            LET COEF=2.0
        OTHERWISE
            LET COEF=4.0
        ALWAYS
        LET FX=FUN.CDF(APARM,BPARM,X)
        LET GN.COMPL=(1.0-FX)**N
        LET FV(1)=GN.COMPL
        LET FV(2)=X*GN.COMPL
        FOR J=1 TO 2, ADD COEF*FV(J) TO SUMV(J)
        IF MOD.F(I,20)=0 AND IPRINT=1 ''PRINT RESULTS
            PRINT 1 LINE WITH X,FX,1.0-GN.COMPL
            THUS
............*.*****    *.*****
```

```
        ALWAYS
        IF GN.COMPL  LT  0.0001 AND COEF = 2.0
            FOR J=1 TO 2, SUBTRACT FV(J) FROM SUMV(J)
            GO TO L1
        OTHERWISE
    LOOP ''OVER I
'L1'IF IPRINT=1
        PRINT 2 LINES THUS
-------------------------------------------------------------------------
    ALWAYS
    LET EMINX=SUMV(1)*DELX/3.0
    LET VMINX=2.0*SUMV(2)*DELX/3.0-EMINX**2
    IF IPRINT=1
        PRINT 2 LINES WITH EMINX,SQRT.F(VMINX)
        THUS
MEAN VALUE OF MIN R.V. IN THE SET _ .............
STANDARD DEV  MIN R.V. IN THE SET _ .............
    ALWAYS
    RELEASE FV(*)
    RELEASE SUMV(*)
    RETURN
END ''MINRV
```

```
ROUTINE TO LIMSTATE ''OF A FINITE, MULTI-SERVER QUEUEING SYSTEM'' GIVEN
LAMBDA, MU, MPOP, NSERVE, PLIM, IPRINT YIELDING LSTATE, P.NULL, P.NSTATE
''
''ROUTINE SOLVES FOR THE PROBABILITY VECTOR WHICH CHARACTERIZES THE STEADY-
''STATE OF A QUEUEING SYSTEM WITH A FINITE POPULATION (MPOP) OF CUSTOMERS
''SERVED BY NSERVE CHANNELS OF EXPONENTIAL SERVICE.  THE ARRIVAL RATE FOR
''EACH INDIVIDUAL NOT IN THE SYSTEM IS A CONSTANT (LAMBDA).  THE SERVICE
''RATE IS THE CONSTANT MU.  THE ROUTINE RETURNS THE (INTEGER) STATE INDEX
''WHICH IS EXCEEDED WITH PROBABILITY PLIM.  THE ROUTINE ALSO RETURNS
''THE PROBABILITY OF AN EMPTY SERVICE SYSTEM (P.NULL) AND THE VECTOR OF
''STATE PROBABILITIES (P.NSTATE(*)).  REF:  GROSS AND HARRIS, FUND. QUEUE.
''
''INPUT:
''LAMBDA            ARRIVAL RATE FOR SERVICE PER INDIVIDUAL.
''MU                SERVICE RATE PER SERVER.
''MPOP              CUSTOMER POPULATION SIZE.
''NSERVE            NUMBER OF SERVICE CHANNELS.
''PLIM              LIMIT ON THE UPPER-TAIL PROBABILITY OF THE STATE
''                  PROBABILITY DISTRIBUTION.
''IPRINT            INTEGER SWITCH TO PRINT FROM THE SUBROUTINE.  PRINTING
''                  OCCURS WHEN IPRINT=1.
''OUTPUT:
''LSTATE            INDEX OF THE MARKOV STATE WHICH IS EXCEEDED WITH
''                  PROBABILITY PLIM.
''P.NULL            PROBABILITY THE SERVICE SYSTEM IS EMPTY.
''P.NSTATE          VECTOR IN WHICH THE N TH ELEMENT IS THE PROBABILITY
''                  THAT N CUSTOMERS ARE IN THE SERVICE SYSTEM.
''
''ENDOGENOUS VARIABLES:
''ENO.DOWN          AVERAGE NUMBER OF INDIVIDUALS IN THE SERVICE SYSTEM.
''SDNO.DOWN         STD DEV NUMBER OF INDIVIDUALS IN THE SERVICE SYSTEM.
''P.SYS.FULL        PROBABILITY THAT ALL SERVICE CHANNELS ARE BUSY.
''E.BUSY.SERVERS    AVERAGE NUMBER OF BUSY SERVICE CHANNELS.
''SD.BUSY.SERVERS   STD DEV NUMBER OF BUSY SERVICE CHANNELS.
''ENO.QUEUED        AVERAGE NUMBER OF UNITS WAITING IN THE SERVICE QUEUE.
''ESYS.WAIT         AVERAGE WAITING TIME IN THE SERVICE SYSTEM.
''EQ.WAIT           AVERAGE WAITING TIME IN THE SERVICE QUEUE, INCLUDING
''                  THE INSTANCES OF ZERO QUEUE TIME.
''
DEFINE I,IPRINT,J,LSTATE,M,MAX,MPOP,N,NSERVE AS INTEGER VARIABLES
DEFINE P.NSTATE AS A REAL, 1-DIMENSIONAL ARRAY
    RESERVE P.NSTATE(*) AS MPOP
    LET R=LAMBDA/MU
    LET C=NSERVE
    IF NSERVE GE MPOP
        PRINT 1 LINE WITH NSERVE,MPOP
        THUS
INPUT ERROR TO ROUTINE LIMSTATE.  NSERVE = *****  MPOP = *****.
        RETURN
    OTHERWISE
    LET CDF.LIM=1.0-PLIM
    LET RHO=R/C
    LET ENO.DOWN=0.0
    LET E.BUSY.SERVERS=0.0
    LET VAR.BUSY.SERVERS=0.0
    LET SUM=1.0
```

```
     LET QSUM=C
     LET NFACTORIAL=1.0
     LET RATIO.FACTORIALS=1.0
     IF NSERVE=1
         GO TO PASS
     OTHERWISE
     FOR N=1 TO NSERVE-1 DO
         LET NFACTORIAL=NFACTORIAL*N
         LET RATIO.FACTORIALS=RATIO.FACTORIALS*(MPOP-N+1)
         LET P.N=R**N/NFACTORIAL*RATIO.FACTORIALS ''OMITTING *P.NULL
         LET P.NSTATE(N)=P.N ''WITH THE NTH STATE STORED IN THE NTH ELEMENT
         ADD N*P.N TO ENO.DOWN
         ADD (NSERVE-N)*P.N TO QSUM
         ADD P.N TO SUM
         ADD N*P.N TO E.BUSY.SERVERS
         ADD N**2*P.N TO VAR.BUSY.SERVERS
     LOOP ''TO DEVELOP SUMS
 ''
 ''CALCULATE THE LAST TERM IN THE SUM FOR P.NULL
 ''
 'PASS'LET CFACTORIAL=C*NFACTORIAL
     LET CEXPOC=C**NSERVE
     LET COEF=CEXPOC/CFACTORIAL
     LET SUM.LAST=0.0 ''SUM FOR PROB THAT NO IN SYSTEM  GT  OR = NSERVE
     LET VNO.DOWN=VAR.BUSY.SERVERS
     FOR N=NSERVE TO MPOP DO
         LET RATIO.FACTORIALS=RATIO.FACTORIALS*(MPOP-N+1)
         LET P.N=COEF*RATIO.FACTORIALS*RHO**N
         ADD P.N TO SUM.LAST ''OMITTING *P.NULL
         ADD N*P.N TO ENO.DOWN ''OMITTING *P.NULL
         ADD N**2*P.N TO VNO.DOWN ''OMITTING *P.NULL
         LET P.NSTATE(N)=P.N ''OMITTING *P.NULL
     LOOP ''TO OBTAIN THE LAST TERM
     ADD SUM.LAST TO SUM
     LET P.NULL=1.0/SUM ''PROB OF SYSTEM NULL STATE
 ''
 ''CHECK OF CALCULATED SERVICE SYSTEM STATE-PROBABILITIES.
 ''
     IF P.NULL GE 1.0 OR P.NULL LE 0.0
         PRINT 1 LINE WITH P.NULL THUS
ERROR IN CALCULATING STATE PROBABILITIES.  P.NULL = .............
         STOP
     OTHERWISE ''CALCULATE EXPECTED VALUES
     LET ENO.DOWN=ENO.DOWN*P.NULL
     LET VNO.DOWN=VNO.DOWN*P.NULL-ENO.DOWN**2
     LET SDNO.DOWN=SQRT.F(VNO.DOWN)
     LET P.SYS.FULL=P.NULL*SUM.LAST
     LET E.BUSY.SERVERS=P.NULL*E.BUSY.SERVERS+C*P.SYS.FULL
     LET VAR.BUSY.SERVERS=P.NULL*VAR.BUSY.SERVERS+C**2*P.SYS.FULL
     -E.BUSY.SERVERS**2
     LET SD.BUSY.SERVERS=SQRT.F(VAR.BUSY.SERVERS)
     LET ENO.QUEUED=ENO.DOWN-NSERVE+P.NULL*QSUM
     LET ESYS.WAIT=ENO.DOWN/LAMBDA/(REAL.F(MPOP)-ENO.DOWN)
 ''ABOVE EXPRESSION IS KNOWN AS LITTLE'S FORMULA.
     LET EQ.WAIT=ESYS.WAIT-1.0/MU
```

```
''
''CALCULATE SERVICE SYSTEM STATE PROBABILITIES.
''
     LET PSUM=P.NULL
     FOR N=1 TO MPOP DO
         LET P.NSTATE(N)=P.NSTATE(N)*P.NULL
         ADD P.NSTATE(N) TO PSUM
         IF PSUM LE CDF.LIM
             LET LSTATE=N
         ALWAYS
     LOOP ''OVER ALL STATES
     IF IPRINT NE 1
         RETURN
     OTHERWISE
     SKIP 2 LINES
     PRINT 2 LINES THUS
STATISTICS FOR THE STEADY STATE OF THE SERVICE SYSTEM

     PRINT 8 LINES WITH ENO.DOWN,SDNO.DOWN,P.SYS.FULL,E.BUSY.SERVERS,
     SD.BUSY.SERVERS,ENO.QUEUED,ESYS.WAIT,EQ.WAIT
     THUS
AVERAGE NUMBER OF INDIVIDUALS IN THE SERVICE SYSTEM __ ***.****
STD DEV NUMBER OF INDIVIDUALS IN THE SERVICE SYSTEM __ ***.****
PROBABILITY THAT ALL SERVICE CHANNELS ARE BUSY _______ *.****
AVERAGE NUMBER OF BUSY SERVICE CHANNELS ______________ *.****
STD DEV NUMBER OF BUSY SERVICE CHANNELS ______________ *.****
AVERAGE NUMBER OF INDIVIDUALS QUEUED FOR SERVICE _____ ***.****
AVERAGE WAITING TIME IN THE SERVICE SYSTEM ___________ ****.***
AVERAGE WAITING TIME IN THE SERVICE QUEUE ____________ ****.***
     SKIP 2 LINES
     RETURN
 END ''LIMSTATE
```